SUPPLY DIVERSIFICATION AND DYNAMIC COORDINATION STRATEGIES

IN OPERATIONS MANAGEMENT

by

Tao Li

APPROVED BY SUPERVISORY COMMITTEE:

Dr. Suresh P. Sethi, Chair

Dr. Jun Zhang

Dr. Özalp Özer

Dr. Shun-Chen Niu

Dr. Ernan E. Haruvy

Dedicated to my family.

SUPPLY DIVERSIFICATION AND DYNAMIC COORDINATION STRATEGIES

IN OPERATIONS MANAGEMENT

by

TAO LI, B.Eng., M.S., M.B.A.

DISSERTATION

Presented to the Faculty of

The University of Texas at Dallas

in Partial Fulfillment

of the Requirements

for the Degree of

DOCTOR OF PHILOSOPHY IN

MANAGEMENT SCIENCE

THE UNIVERSITY OF TEXAS AT DALLAS

August 2012

ACKNOWLEDGMENTS

Many people have contributed to the production of this dissertation. Without their help, support and encouragement, this dissertation would not be possible. I owe my gratitude to these people and because of them my graduate experience has been one that I will cherish forever.

I would like to express my deepest appreciation and sincerest gratitude to my advisor, Professor Suresh P. Sethi, who supervised me throughout my doctoral studies and guided me through this journey. Words cannot express my appreciation for his continuous and invaluable help, patience, and support. It is he who opened my eyes to the academic world and led me to an academic career. His influence will continue to benefit me for the rest of my life.

I would also like to express my great appreciation to Professor Jun Zhang for always being patient and available to discuss my research work, for valuable suggestions through my doctoral study, and for his continuous support and encouragement in my career search. It has been my great pleasure working with him during the past five years.

I am deeply grateful to Professor Ernan E. Haruvy for spending hours discussing with me, raising questions and challenging me to expand my thinking, encouraging the use of correct grammar and consistent notation in my writing, and carefully reading and commenting on countless revisions of my research work.

I am also grateful to Professor Milind Dawande who taught me combinatorial optimizations and mathematical programming, and greatly helped me during my job-search process. Interesting discussions with him have always been memorable and inspiring.

I am also thankful to Professor Shun-Chen Niu for leading me to the stochastic world, for always being patient and available to answer my questions, and for serving on my dissertation committee.

I want to thank Professor Özalp Özer for his advice and encouragement during my job-search process and for serving on my dissertation committee.

I would like to extend my thanks to Professor Metin Çakanyıldırım, Professor Ganesh Janakiraman, Professor Chelliah Sriskandarajah, Professor Kathryn E. Stecke, Professor Alain Bensoussan for teaching me excellent courses and helping me during my job-search process.

My gratitude is also extended to Lindsay Hamn, Sophia Orme, Amanda Besch, and Barbara Gorden for their help during the past years.

Many friends have helped me through this journey. I would like to offer my deep thanks to my friend Xiuli He. She has always supported and encouraged me since the very beginning of this journey. I am also grateful to Qi (Annabelle) Feng, Ruixia (Sandy) Shi, Tharanga Rajapakshe, Mili Mehrotra, Jun Ru, and Anshuman Chutani for their help along the way. I appreciate the friendship of Yihua Zhao, Qingning Cao, Chao Liang, Meng Li, and Liying Mu who made my life memorable and pleasurable.

Most importantly, I am very grateful to my family, who I can always count on. I deeply appreciate the unconditional love and continuous support from my parents, Mr. Jintang Li and Mrs. Aihua Feng. Finally, I would like to express my love and appreciation to my wife, Juan Zhao. I appreciate her love, support, and sacrifice and I cannot imagine going through this journey without her by my side. To them, I dedicate this dissertation.

June 2012

PREFACE

This dissertation was produced in accordance with guidelines which permit the inclusion as part of the dissertation the text of an original paper or papers submitted for publication. The dissertation must still conform to all other requirements explained in the "Guide for the Preparation of Master's Theses and Doctoral Dissertations at The University of Texas at Dallas". It must include a comprehensive abstract, a full introduction and literature review and a final overall conclusion. Additional material (procedural and design data as well as descriptions of equipment) must be provided in sufficient detail to allow a clear and precise judgment to be made of the importance and originality of the research reported. It is acceptable for this dissertation to include as chapters authentic copies of papers already published, provided these meet type size, margin and legibility requirements. In such cases, connecting texts which provide logical bridges between different manuscripts are mandatory. Where the student is not the sole author of a manuscript, the student is required to make an explicit statement in the introductory material to that manuscript describing the student's contribution to the work and acknowledging the contribution of the other author(s). The signatures of the Supervising Committee which precede all other material in the dissertation attest to the accuracy of this statement.

SUPPLY DIVERSIFICATION AND DYNAMIC COORDINATION STRATEGIES

IN OPERATIONS MANAGEMENT

Publication No. ____________________

Tao Li, Ph.D.
The University of Texas at Dallas, 2012

Supervising Professor: Dr. Suresh P. Sethi

This dissertation addresses three important problems in manufacturing and service operations management.

In Chapter 2, we study sourcing and pricing decisions of a firm with correlated suppliers and a price-dependent demand. With two suppliers, the insight—*cost is the order qualifier while reliability is the order winner*—derived in the literature for the case of exogenously determined price and independent suppliers, continues to hold when the suppliers' capacities are correlated. Moreover, a firm orders only from one supplier if the effective purchase cost from him, which includes the imputed cost of his unreliability, is lower than the wholesale price charged by his rival. Otherwise, the firm orders from both. Furthermore, the firm's diversification decision does *not* depend on the correlation between the two suppliers' random capacities. However, its total order quantity decreases as the capacity correlation increases in the sense of the supermodular order. With more than two suppliers, the insight no longer holds. That is, when ordering from two or more suppliers, one is the lowest-cost supplier and the others are not selected on the basis of their costs. We also develop a solution algorithm for the firm's optimal diversification problem.

In Chapter 3, we study sourcing decisions of price-setting and price-taking firms with two unreliable suppliers, where a price-setting firm sets the retail price and a price-taking firm takes the retail price as given. We investigate the impacts of market conditions, suppliers' wholesale prices and their reliabilities on the optimal sourcing decisions of price-setting and price-taking firms, and examine how a firm's pricing power affects these impacts. We define a supplier's reliability in terms of the "size" or the "variability" of his random capacity using the concepts of stochastic dominance. We find that the supplier reliability affects the optimal sourcing decisions differently for price-setting and price-taking firms. Specifically, with a price-setting firm, a supplier can win a larger order by increasing his reliability, it is not always so with a price-taking firm.

In Chapter 4, we consider a supply chain in which the manufacturer has two production and sales opportunities and sells the product to a retailer. The manufacturer's second-period production cost declines linearly in the first-period production quantity. We show that as the production cost learning process becomes more efficient and less stable, the traditional double marginalization problem becomes more severe. This leads to a greater efficiency loss in the decentralized channel. We propose a two-period revenue sharing contracts to coordinate the two-period dynamic supply chain. We also investigate the implications of the learning curve on the value of strategic inventory.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

1.1 Supply Diversification with Responsive Pricing

Supply diversification or multiple sourcing has been demonstrated to be an effective strategy for firms to ward against potential supply disruptions due to natural disasters, random yields, quality issues, delivery uncertainties, supplier bankruptcies, etc. Firms not diversifying properly may incur significant financial losses from supply disruptions. In 2000, when a fire occurred at a semiconductor plant in New Mexico, which was the primary source for one of the key electronic chips for Ericsson and Nokia, Ericsson lost at least $400 million in potential revenue as a result of not having a "Plan B", while Nokia was able to source from alternative suppliers to avoid substantial financial losses (Latour 2001). Forty-two days after the March 11 (2011) earthquake/tsunami struck Japan, Toyota said it still faces critical shortages for about 150 parts which already dropped from roughly 500 parts in mid-March. Executive Vice President Shinichi Sasaki said Toyota will reassess its dependence on single-source suppliers in Japan (Dawson and Takahashi 2011). For those firms that Toyota sourced from, it is important to implement supply diversification to avoid substantial losses of business.

The extant literature on supply diversification largely assumes that the suppliers' capacities are independent for analytical tractability. Under this assumption, it is well known that "cost is the order qualifier while reliability is the order winner" ("cost first reliability second" for short). However, supplier independence does not often hold in reality. The suppliers' capacities are correlated because suppliers are linked through their industries, customers, suppliers, geographic proximity, and national economies. For example, most suppliers in Japan were affected by disruptions caused by the earthquake mentioned above. It is very likely in this situation that their capacities are positively correlated (Clark and Takahashi 2011). Capacity dependence among different suppliers may also occur when the suppliers' total capacity is limited by a quota set by a central government. When China reduces its export quota for rare-earth elements (REE), capacities of all Chinese REE suppliers will be affected (WSJ 2010), resulting in a positive correlation among the suppliers. Negatively correlated capacities among suppliers may happen when suppliers have to compete for the same critical resource/component for their operations. It would be interesting to see if the cost-first-reliability-second insight can be extended to cases where suppliers capacities are dependent.

Another strategy that firms adopt to deal with supply uncertainties is *responsive pricing*, i.e., firms determine retail prices after supply uncertainties are resolved. Dell responds to supply disruptions in memory cards via shifting customer demand to lower-memory personal computers by adjusting prices (Tomlin 2006). In the semiconductor and electronic component industries, many manufacturers respond to supply disruptions by adjusting prices based on, among other factors, the availability of inventory on hand (Li and Zheng 2006). It is natural that firms will raise the retail prices when they encounter shortages. For instance, spot prices of the 4-gigabit NAND flash memories that find their way into smartphones, tablet computers, and digital cameras rose 17 percent on March 14, 2011 after the earthquake struck Japan (Einhorn et al. 2011). Therefore, it is important to study the firm's supply diversification problem with responsive pricing.

1.2 How does Pricing Power Affect A Firm's Sourcing Decisions from Unreliable Suppliers?

The benefit of multiple sourcing or supply diversification is well established in mitigating supply risks arising from supply disruptions caused by natural disasters, random yields/capacities, delivery uncertainties, quality issues, suppliers declaring bankruptcy, etc. An example is the March 11, 2011 earthquake in Japan that totally disrupted the global supply chains of some prominent firms (Dawson and Takahashi 2011). According to Shinichi Sasaki, the Executive Vice President of Toyota, "Even in cases where we thought we had more than one supplier, it turned out in many cases that they procured subcomponents from just one firm." That is, Toyota would have liked its supplier firms to be multi-sourcing. Indeed, according to (Dawson 2011), the post-quake crisis has prompted auto makers to seek ways to diversify their supply chains for critical components. See also Tomlin (2006) and Federgruen and Yang (2009) for other examples of supply diversification.

In this Chapter, we study how pricing power affects a firm's sourcing decisions from unreliable suppliers. A firm's pricing power is its ability to adjust the market price of a good or service. A firm's pricing power depends on a number of factors including its product's uniqueness in the marketplace, competition from similar products, consumer perception of the quality of the product, and the effectiveness of the company's advertisement campaign. In a perfectly competitive market, a firm has no pricing power, whereas a monopoly can set its price to maximize its profit. In this work, we study the sourcing decisions of price-setting and price-taking firms from unreliable suppliers and examine the impacts of their different pricing powers by comparing their sourcing strategies. For our purposes, a price-setting firm is one that sets the retail price after supply uncertainty is resolved, whereas a price-taking firm sells its product at an exogenously given retail price.

1.3 Strategic Inventories and Dynamic Coordination with Production Cost Learning

The phenomena of production learning, i.e., the decline of production cost with cumulative production has been well observed in the last few decades and in a number of industries such as aircraft manufacturing, automobile assembly, apparel manufacturing, production of large musical instruments, and semiconductor manufacturing (see, for example, Wright 1936, Baloff 1971, Yelle 1979, Hatch and Mowery 1998). Operations managers have used learning curve as a strategic tool for capacity planning decisions (Mazzola and McCardle

1997). Most existing research on this subject focuses only on deriving the optimal production quantity from the manufacturer's perspective. However, under make-to-order policy in a decentralized channel, the manufacturer sells the product through an independent retailer and produces according to the retailer's order. The retailer's decisions will certainly affect the manufacturer's production decisions. Furthermore, if both the manufacturer and the retailer have market power to set prices, their pricing strategies will greatly impact the demand of the product and therefore the manufacturer's production decisions.

Inventory may be carried for operational reasons such as economy of scale in production, delivery and production lead time, uncertain consumer demand and manufacturing capacity constraints. Besides these classical reasons for holding inventories, previous research has analyzed the strategic roles that inventories can play in both horizonal and vertical competitions. For example, inventories serve as a commitment to achieve the first-mover advantage (Saloner 1986) and deter deviation from collusive profits (Rotemberg and Saloner 1989). In a vertical supply chain with a buyer and a supplier, the buyer carries strategic inventories from the first period to force the supplier to offer favorable wholesale price terms in the second period (Anand et al. 2008). Strategic inventory can effectively alleviate the well-known double marginalization effect[1] Therefore, it can be used to improve the channel efficiency. In this chapter, we investigate how the value of strategic inventory is affected in the presence of production cost learning.

1.4 Organization of the Dissertation

In Chapter 2, we study sourcing and pricing decisions of a firm with unreliable dependent suppliers and a price-dependent demand. We investigate whether the insight—*cost is the order qualifier while reliability is the order winner*—derived in the literature for the case of exogenously determined price and independent suppliers continues to hold when the

[1]Double marginalization is a well-known problem in a decentralized supply chain. When the supplier and the buyer maximize their individual profits, the buyer and supplier add their margins to the price quoted downstream and the final retail price is higher than the systematic optimal level.

suppliers' capacities are correlated. Moreover, we examine how the firm's diversification decisions depend on the capacity correlation.

In Chapter 3, we study sourcing decisions of price-setting and price-taking firms with two unreliable independent suppliers. We investigate the impacts of market conditions, suppliers' wholesale prices and their reliabilities on the optimal sourcing decisions of price-setting and price-taking firms, and examine how a firm's pricing power affects these impacts.

In Chapter 4, we consider a decentralized supply chain in which the manufacturer has two production and sales opportunities and sells the product to an independent retailer. The second-period production cost declines linearly in the first-period production quantity with some randomness in the learning rate. We investigate the implications of the learning curve on the value of strategic inventory and dynamic channel coordination.

Li et al. (2012a) is based on Chapter 2. The main results in Li et al. (2012b) are from Chapter 3 and the main results in He et al. (2012) are based on Chapter 4. I would like to express my sincere appreciation to my co-authors, Professors Suresh P. Sethi, Jun Zhang, and Xiuli He for their invaluable contributions.

CHAPTER 2

SUPPLY DIVERSIFICATION WITH RESPONSIVE PRICING

2.1 Synopsis

We study sourcing and pricing decisions of a firm with unreliable dependent suppliers and a price-dependent demand. By an *unreliable* supplier, we mean that the supplier's capacity (maximum quantity he can deliver) is a random variable. Our analysis demonstrates that with two dependent suppliers, the cost-first-reliability-second insight in picking suppliers continues to hold. Whether a firm purchases from a supplier depends on his wholesale price and the effective purchase cost from his rival, where the effective purchase cost from a supplier is defined to be his wholesale price plus the imputed cost of his unreliability. Specifically, the firm always orders from the supplier with the lower wholesale price. If the effective purchase cost from this supplier is less than his rival's wholesale price, the firm orders only from this supplier. Otherwise, the firm should order from both.

We demonstrate that with two suppliers, the firm's diversification decision depends on the suppliers' random capacities only through their marginal distributions. In other words, the correlation structure between the suppliers' capacities has no bearing on the firm's diversification decisions. Nevertheless, the correlation structure does affect the firm's optimal order quantities when it chooses to diversify. As the suppliers' capacities become more correlated in the sense of the supermodular order, the firm's optimal total order quantity decreases. Furthermore, when both suppliers' wholesale prices are low, the optimal order quantity for each supplier decreases as the capacity correlation increases in the supermodular order.

With more than two suppliers, the insight "cost first reliability second" no longer holds. That is, when ordering from two or more suppliers, one is the lowest-cost supplier and the others are not selected on the basis of their costs. Specifically, a low wholesale price does not guarantee an order from the firm, although the lowest wholesale price does. Moreover, if a supplier's wholesale price is greater than the firm's effective purchase cost from any group of suppliers, the firm never orders from that supplier. We show that, in addition to cost and reliability, the firm must consider the capacity dependence among selected suppliers when deciding whether or not to order from an additional supplier. Doing so may improve the firm's profit by as much as 5% as demonstrated by an example.

In general, solving the firm's sourcing and pricing problem with dependent suppliers is extremely involved. To solve the problem, we extend the concept of the effective purchase cost from a supplier to a group of suppliers. Then, we establish three key structural properties of the problem using the extended concept: First, the firm should always source from the lowest-wholesale-price supplier. Second, the firm should never source from a supplier whose wholesale price is greater than the effective purchase cost from any group of suppliers. Third, changes in supplier capacity correlation does not affect the firm's diversification solution as long as the correlation remains "positive". Moreover, we develop a solution algorithm for the problem based on these properties.

The remainder of this chapter is organized as follows. In section 2.2 we review the related literature. Section 2.3 introduces our model with several basic assumptions. In section 2.4 we analyze and solve the problem with two suppliers. In section 2.5 we study the case with more than two suppliers. We conclude this chapter with discussion of the results in section 2.6.

2.2 Literature Review

This chapter is related to three streams of literature. One stream deals with procurement strategies under unreliable supply while price is commonly assumed to be exogenous. Most studies examine inventory decisions with random yield; see, for example, Yano and Lee (1995) and Grosfeld-Nir and Gerchak (2004) for extensive reviews of the literature. Ciarallo et al. (1994), on the other hand, analyze random supply capacity, and they demonstrate that in the presence of capacity uncertainty, a base-stock policy remains optimal. While these papers focus on the procurement/production decisions with one unreliable supplier, this chapter examine the firm's diversification and ordering decisions with multiple unreliable suppliers.

The advantage of supply diversification in the presence of supply uncertainty is well established in the literature (Gerchak and Parlar 1990; Ramasesh et al. 1991; Parlar and Wang 1993). Recent studies of a dual sourcing problem with one unreliable and one perfectly reliable supply source include Kazaz (2004), Tomlin (2006), and Tomlin and Snyder (2007). The scenario with multiple unreliable suppliers is investigated in Agrawal and Nahmias (1997), Tomlin and Wang (2005), Tomlin (2009), and Wang et al. (2010).

With two or more unreliable suppliers, whose random capacities are independent, it has been established that cost always takes precedence over reliability when it comes to selecting suppliers, and reliability affects the order quantity from a selected supplier. In other words, *cost is the order qualifier and reliability is the order winner*. Anupindi and Akella (1993) demonstrate that it is always optimal to order some amount from the less expensive supplier, when the initial inventory is insufficient in multi-period settings where the supply uncertainty can be either in delivery time or delivery quantity or both. Dada et al. (2007) establish the cost-first-reliability-second insight in a single-period setting with more general assumptions on supply uncertainty, where the firm pays for the delivered quantity. Federgruen and Yang (2009) demonstrate a result similar to Dada et al. (2007) in two versions of the planning model—the service constraint model and the total cost model—in a setting where the firm pays for each unit ordered. Burke et al. (2009) also demonstrate a similar insight as in Dada et al. (2007). Swaminathan and Shanthikumar (1999), on the other hand, find that in the case of discrete demand, ordering from the more expensive supplier alone may be optimal.

We study a supply diversification problem with correlated suppliers and responsive pricing. Incorporating supply dependence significantly changes the insight on the firm's supply diversification decision. In particular, while the insight—cost is the order qualifier and reliability is the order winner—continues to hold with two suppliers, it does not hold with more than two suppliers. A firm must allow for the correlation structure between the suppliers' capacities when deciding on the optimal supplier base, and should not rank suppliers purely based on their wholesale prices in its sourcing decision.

The second stream of literature related to this chapter focuses on inventory models with price-dependent demands. Most of the operations management literature, dealing with pricing in inventory/capacity management, focuses on a single product with perfectly reliable supply. The initial work on endogenous pricing in inventory/capacity models was done by Whitin (1955) and Mills (1959, 1962). Comprehensive reviews of the newsvendor-type models with endogenous pricing are provided by Porteus (1990) and Petruzzi and Dada (1999). Boyaci and Ozer (2010) studies the effect of pricing and information acquisition in production/capacity planning problems. Li and Zheng (2006) address joint pricing and inventory decisions in the presence of yield uncertainty, and show that the optimal inventory replenishment is characterized by a threshold value. Feng (2010), on the other hand, investigates dynamic pricing and replenishment decisions in the presence of supply capacity uncertainty, and show that the base-stock list-price policy fails to achieve optimality even under deterministic demand.

Only a few papers have studied a firm's sourcing problem with multiple unreliable suppliers and a price-dependent demand. Tang and Yin (2007) study the benefits of responsive pricing for a firm with supply uncertainty and address the issue of order allocation among multiple unreliable suppliers. Feng and Shi (2012) study dynamic responsive pricing with multiple suppliers, random capacity, and stochastic linear demand. They demonstrate that the cost-first-reliability-second insight is no longer valid for general random capacity distributions when price can be adjusted dynamically. Li et al. (2012b) study the impact of pricing power on a firm's sourcing decisions. One major difference between our work and the previous research is that we assume the suppliers' capacities to be dependent. We focus on the impact of supplier correlation on a firm's sourcing decisions in this chapter.

The third stream of literature related to this chapter focuses on the effects of supplier correlation. While Dada et al. (2007) and Federgruen and Yang (2009) discuss how their results might be extended to cases with correlated supply capacities, only Babich et al. (2007), to the best of our knowledge, have studied the impact of supplier correlation on a firm's sourcing decision. They demonstrate that a firm may benefit from high supplier default correlations because low supplier default correlations may dampen competition among the suppliers. We do not consider the strategic interaction between the suppliers and the firm; however, we model the supplier dependence in a more general way. Moreover, we consider the firm's demand to be price dependent.

2.3 Model

Consider a risk-neutral firm that may order parts from N suppliers and sells products using those parts in the market in a single selling season. The firm uses the strategy of *responsive pricing*, i.e., it determines the retail price after it knows the quantity of the delivered parts. The suppliers differ from one another in terms of their capacity distributions as well as their wholesale prices. If a supplier can meet fully the firm's order regardless of the order size, then he is *perfectly reliable*. Otherwise, he is unreliable. Supplier i ($i = 1, 2, \ldots, N$), when unreliable, has a random capacity R_i that is exogenously given; if supplier i is perfectly reliable, then we have the special deterministic case with $R_i \equiv \infty$. We assume that R_i has the cumulative distribution function (cdf) $G_i(r)$ and the corresponding probability density function (pdf) $g_i(r) \geqslant 0$ for $r > 0$. Let $\overline{G}_i(r) \equiv 1 - G_i(r)$. If the suppliers' capacities are dependent, then their capacities are characterized by a joint probability density function $g(r_1, r_2, \ldots, r_N)$.

The selling season consists of two stages. In the first stage, the firm orders a quantity Q_i from supplier i at the wholesale price of c_i, and receives the quantity $S_i(Q_i) = \min\{Q_i, R_i\}$. Denote $\mathbf{Q} \equiv (Q_1, Q_2, \ldots, Q_N)$ as the vector of order quantities. Let $Q = Q_1 + \cdots + Q_N$ denote the total order quantity and $S(\mathbf{Q}) = S_1(Q_1) + \cdots + S_N(Q_N)$ denote the total delivered quantity. The firm pays a supplier only for the quantity delivered. In the second stage, based on the total delivered quantity $S(\mathbf{Q})$, the firm decides the unit retail price p for the product. We assume the demand to be deterministic and price-dependent according to an additive form, that is, $D(p) = a - bp$, where $a > 0$ is the potential market size and $b > 0$ is the price sensitivity of the demand. To ensure that the firm is able to make a positive profit and avoid trivial cases, we assume that $c_i < a/b$. This chapter assumes holdback rather than clearance, and thus, there could be unsold units. The firm may salvage unsold products in a secondary market at a unit price $\gamma < c_i$. The cost of lost goodwill is δ for each unit of unfulfilled demand.

The firm's objective is to choose the order quantities $\mathbf{Q}$ in the first stage and the retail price p in the second stage to maximize its expected profit $\Pi(\mathbf{Q})$, which is equal to its expected second-stage profit $E[\Pi_2(\mathbf{Q})]$ less its expected purchase cost in the first stage. The firm's problem is:

$$\max_{\mathbf{Q}\geqslant 0}\left\{\Pi(\mathbf{Q}) \;=\; E\left[\Pi_2(\mathbf{Q}) - \sum_{i=1}^{N} c_i S_i(Q_i)\right]\right\}, \tag{2.1}$$

$$\begin{aligned}\text{where}\quad \Pi_2(\mathbf{Q}) &= \max_{p\geqslant 0}\pi(p) = \max_{p\geqslant 0}\{p\cdot\min\{D(p), S(\mathbf{Q})\} \\ &\quad +\gamma\cdot(S(\mathbf{Q}) - D(p))^{+} - \delta\cdot(D(p) - S(\mathbf{Q}))^{+}\}.\end{aligned} \tag{2.2}$$

In this formulation, the firm's second-stage profit is equal to the sum of its sales and salvage revenues if there are leftover products, or equal to its sales revenue less the shortage cost if there are shortages. Before we proceed, we first solve the firm's second-stage problem to obtain the retail price p that maximizes its profit for a given total supply S. From (2.2), we know that the firm's second-stage problem can be formulated as:

$$\max_{p\geqslant 0}\left\{\pi(p) \;=\; p\cdot min\{D(p), S\} + \gamma\cdot(S - D(p))^{+} - \delta\cdot(D(p) - S)^{+}\right\}. \tag{2.3}$$

Theorem 2.3.1 *For a given S, the optimal retail price is*

$$p^* = \begin{cases} \dfrac{a+\gamma b}{2b}, & \textit{if } S \geqslant \dfrac{a-\gamma b}{2}, \\ \dfrac{a-S}{b}, & \textit{otherwise.} \end{cases}$$

Proof of Theorem 2.3.1. The proof is based on the easily provable facts that $\pi(p)$ is increasing for $p < (a-S)/b$, and $(a+\gamma b)/2b$ is the solution of $\pi'(p) = 0$ with $\pi''(p) < 0$, $p \geqslant (a-S)/b$. ■

That is, the firm sets the price so as to sell all when the total delivery is less than $(a-\gamma b)/2$. Otherwise, the firm sets the price at $(a+\gamma b)/2b$ and salvages the leftover products. Since any quantity delivered in excess of $A \equiv (a-\gamma b)/2$ will be salvaged, we refer to A as the *abundant supply*. Throughout this chapter, we will utilize the marginal analysis to better understand the firm's optimal decision. We denote MR as the marginal revenue and MC as the marginal cost. By Theorem 2.3.1, for a given total delivery quantity S, the marginal revenue is

$$\mathrm{MR} = \begin{cases} \gamma, & \text{if } S \geqslant A, \\ \dfrac{a-2S}{b}, & \text{otherwise.} \end{cases} \tag{2.4}$$

Note that when the delivery quantity is greater than the abundant supply, the additional unit will be salvaged at γ; consequently, the marginal revenue is γ.

We proceed now to the firm's first-stage problem. Let $f_{\mathbf{Q}}(s)$ be the density of the random variable S given $\mathbf{Q}$. By (2.1) and Theorem 2.3.1, the firm's first-stage problem can be formulated as:

$$\begin{aligned} \max_{\mathbf{Q} \geqslant 0} \Big\{ \Pi(\mathbf{Q}) \;\; &= \;\; \int_0^A \frac{s(a-s)}{b} f_{\mathbf{Q}}(s) ds + \int_A^\infty \left[\frac{a^2 - (\gamma b)^2}{4b} + \frac{\gamma(2s - a + \gamma b)}{2} \right] f_{\mathbf{Q}}(s) ds \\ &\quad - \sum_{i=1}^N c_i \left[\int_0^{Q_i} r dG_i(r) + Q_i \overline{G}_i(Q_i) \right]. \end{aligned} \tag{2.5}$$

On the right-hand side of (2.5), the first term is the expected sales revenue when the total delivery is less than the abundant supply. The second term is the expected sales revenue plus the salvage revenue when the total delivery is greater than the abundant supply. The last term is the total expected purchase cost.

2.4 Two Suppliers

We first solve the single unreliable supplier problem as a building block. Let $i = 1$ be the only unreliable supplier. Obviously, it is never optimal to order more than the abundant supply. When the order quantity is less than the abundant supply, the marginal revenue $\mathrm{MR} = (a - 2Q_1)\overline{G}_1(Q_1)/b$ and the marginal cost $\mathrm{MC} = c_1\overline{G}_1(Q_1)$. Note that both the marginal revenue and the marginal cost are adjusted by the term $\overline{G}_1(Q_1)$. Intuitively, only when the random capacity is greater than the order quantity, does an increase in the order quantity yield a positive marginal revenue and a positive marginal cost; otherwise, both marginals are zero. By setting MR=MC, we easily see that when the firm orders only from supplier $i = 1$, the optimal order quantity is $(a - bc_1)/2$. This result is consistent with Ciarallo et al. (1994) in that the optimal order quantity does not depend on the supplier's reliability.

In order to solve the two-supplier problem (now $i = 1, 2$), we divide the positive (Q_1, Q_2) quadrant into five regions, according to the expressions for the expected profit function in (2.5). These regions are: Region I: $Q_1 \geqslant 0, Q_2 \geqslant 0, Q_1 + Q_2 \leqslant A$; Region II: $Q_1 + Q_2 > A, Q_1 \leqslant A, Q_2 \leqslant A$; Region III: $0 \leqslant Q_1 \leqslant A, Q_2 > A$; Region IV: $0 \leqslant Q_2 \leqslant A, Q_1 > A$; Region V: $Q_1 > A, Q_2 > A$. From the fact that the capacities are unreliable and Theorem 2.3.1, clearly, the optimal order quantities fall in either Region I or Region II. Consequently, finding the firm's optimal ordering decisions requires us to solve problem (2.5) in these two regions and then select the best.

Denote $(\overline{Q}_1, \overline{Q}_2)$ as the solutions to the first-order condition for maximizing the profit function in Region I:

$$[a - bc_i - 2\,(Q_1 + Q_2)]\,\overline{G}_i(Q_i) + 2\int_0^{Q_{3-i}} \int_{Q_i}^{\infty} (Q_{3-i} - r_{3-i}) g(r_1, r_2) dr_i dr_{3-i} = 0, \;\; \text{for } i = 1, 2. \tag{2.6}$$

Equation (2.6) can be interpreted by using marginal analysis. In Region I, if supplier i's random capacity r_i turns out to be less than his order quantity Q_i, then the firm's marginal revenue and cost from this supplier are both zero. On the other hand, conditional on the full delivery of supplier i, the firm's marginal cost from supplier i is c_i, and its marginal revenue from supplier i depends on the realization of supplier $(3-i)$'s capacity r_{3-i}. If the random capacity turns out to be greater than the order quantity for supplier $(3-i)$, then the total delivery is Q_1+Q_2 and the marginal revenue is $(a-2(Q_1+Q_2))/b$; otherwise, the total delivery is Q_i+r_{3-i} and the marginal revenue is $(a-2(Q_i+r_{3-i}))/b$. Equating the expected marginal revenue and the expected marginal cost yields equation (2.6). Denote $\widehat{Q}_i$ $(i=1,2)$ as the solutions to the first-order condition for maximizing the profit function in Region II:

$$\int_{Q_i}^{\infty}\int_0^{A-Q_i} 2\left(A-(Q_i+r_{3-i})\right)g(r_1,r_2)dr_{3-i}dr_i - b(c_i-\gamma)\overline{G}_i(Q_i)=0, \text{ for } i=1,2. \quad (2.7)$$

Equation (2.7) can also be interpreted by using marginal analysis. In Region II, if supplier i's random capacity r_i turns out to be less than Q_i, then the firm's marginal revenue and cost are both zero. On the other hand, conditional on the full delivery of supplier i, the firm's marginal cost is c_i, and the firm's marginal revenue depends on the realization of supplier $(3-i)$'s capacity r_{3-i}. If the random capacity of supplier $(3-i)$ turns out to be larger than $A-Q_i$, then the total delivery will exceed the abundant supply and the marginal revenue will be γ; otherwise, the total delivery is Q_i+r_{3-i} and the marginal revenue will be $(a-2(Q_i+r_{3-i}))/b$. Equating the expected marginal revenue and the expected marginal cost yields equation (2.7).

Define $h_i(\cdot)=g_i(\cdot)/(1-G_i(\cdot))$ as the hazard rate function of R_i. To ensure that the profit function is unimodal, we make some assumptions specified in the following lemma.

Lemma 2.4.1 *For $i=1,2$, assume that $(\overline{Q}_1,\overline{Q}_2)$ satisfies the unimodality conditions*

$$\int_0^{\overline{Q}_{3-i}}\left\{(\overline{Q}_{3-i}-r_{3-i})g(\mathbf{x^i})+\int_{\overline{Q}_i}^{\infty}[1-h_i(\overline{Q}_i)(\overline{Q}_{3-i}-r_{3-i})]g(r_1,r_2)dr_i\right\}dr_{3-i}\geqslant 0 \quad (2.8)$$

and that $(\widehat{Q}_1,\widehat{Q}_2)$ satisfies the unimodality conditions

$$\int_0^{A-\widehat{Q}_i}\left\{\left(A-\widehat{Q}_i-r_j\right)g(\mathbf{x^i})+\int_{\widehat{Q}_i}^{\infty}\left[1-h_i(\widehat{Q}_i)\left(A-\widehat{Q}_i-r_j\right)\right]g(r_1,r_2)dr_i\right\}dr_j\geqslant 0, (2.9)$$

where $\mathbf{x^1} = (\widehat{Q}_1, r_2)$ *and* $\mathbf{x^2} = (r_1, \widehat{Q}_2)$*. Then the profit function in* (2.5) *is unimodal.*

Proof of Lemma 2.4.1. The proof can be found in the proof of Theorem 2.4.3. ■

The unimodality conditions in Lemma 2.4.1 ensure the unimodality of the firm's expected profit function. Verifying these conditions requires solving the first-order conditions (2.6) and (2.7). Next, we provide several sufficient conditions, under which the unimodality conditions hold *a priori*. Before we proceed, we first present some concepts used in multivariate analysis.

For a univariate distribution function F with density f and survival function $\overline{F} = 1 - F$, the ratio $h = f/\overline{F}$ is known as the hazard rate. For a random vector $(T_1, ..., T_n)$, define the joint survival function $\overline{F}$ as $\overline{F}(\mathbf{t}) = P\{T_1 > t_1, ..., T_n > t_n\}$, where $\mathbf{t} = (t_1, ..., t_n)$. The hazard function is defined as $R = -\log \overline{F}$. If R has a gradient $\overline{h} = \nabla R$, we call $\overline{h}$ the hazard gradient. Note that $\overline{h}_i(\mathbf{t})$ can be interpreted as the conditional hazard rate of T_i evaluated at t_i, given $T_j > t_j$ for all $j \neq i$. That is, $\overline{h}_i(\mathbf{t}) = f_i(t_i \mid T_j > t_j, j \neq i)/\overline{F}_i(t_i \mid T_j > t_j, j \neq i)$, where $f_i(\cdot \mid T_j > t_j, j \neq i)$ and $\overline{F}_i(\cdot \mid T_j > t_j, j \neq i)$ are, respectively, the conditional density and survival functions of T_i, given $T_j > t_j$ for all $j \neq i$. Refer to Johnson and Kotz (1975) and Marshall (1975) for details.

Random variables $T_1, ..., T_n$ are *associated* if $Cov[f(\mathbf{T}), g(\mathbf{T})] \geqslant 0$ for all nondecreasing functions f and g for which $Ef(\mathbf{T})$, $Eg(\mathbf{T})$, $Ef(\mathbf{T})g(\mathbf{T})$ exist, where $\mathbf{T} = (T_1, ..., T_n)$ (Esary et al. 1967). The notion of association among random variables is just one among many notions of multivariate dependence. Next, we present several alternative notions of positive dependence which imply association. In application, it is often easier to verify one of these alternative notions. For our purpose, we present the definitions in the bivariate case only; for more in-depth discussions on these concepts, see Barlow and Proschan (1975, Sec. 5.4). Given random variable S and T, (a) T is right tail increasing in S, i.e. $RTI(T \mid S)$, if $P[T > t \mid S > s]$ is increasing in s for all t; (b) S and T are right-corner-set increasing, i.e., $RCSI(S, T)$, if $P[S > s, T > t \mid S > s', T > t']$ is increasing in s' and t' for each fixed s, t; (c) S and T are $TP_2(S, T)$, if the joint probability density $f(s, t)$ is totally positive of order

2, that is, $f(s_1,t_1)f(s_2,t_2) \geqslant f(s_1,t_2)f(s_2,t_1)$ for all $s_1 < s_2$, $t_1 < t_2$ in the domain of S and T. With these preliminary concepts, we present Lemma 2.4.2, which provides several sufficient conditions under which the unimodality conditions in Lemma 2.4.1 hold.

Lemma 2.4.2 *A bivariate capacity distribution satisfies the unimodality conditions* (2.8) *and* (2.9) *if any of the following conditions is satisfied:*

(i) *For* $i = 1, 2$, *any* $Q_i \in [0, A]$ *satisfies*

$$\int_0^{Q_{3-i}} \left\{ (Q_{3-i} - r_{3-i}) g(\mathbf{x^i}) + \int_{Q_i}^{\infty} [1 - h_i(Q_i)(Q_{3-i} - r_{3-i})] g(r_1, r_2) dr_i \right\} dr_{3-i} \geqslant 0,$$

where $\mathbf{x^1} = (Q_1, r_2)$ *and* $\mathbf{x^2} = (r_1, Q_2)$.

(ii) *For* $i = 1, 2$, $h_i(Q_i) + \overline{h}_{3-i}(Q_1, Q_2) \geqslant \overline{h}_i(Q_1, Q_2)$.

(iii) *For* $i = 1, 2$, *the distribution satisfies* $RTI(R_i \mid R_{3-i})$.

(iv) *The distribution satisfies* $RCSI(R_1, R_2)$.

(v) *The distribution satisfies* $TP_2(R_1, R_2)$.

(vi) *The distribution is a bivariate normal distribution with* $\rho \geqslant 0$.

The conditions become stronger as we go down the list.

Proof of Lemma 2.4.2. (i)$\Rightarrow$ unimodality conditions: By the proof of Theorem 2.4.3, any (Q_1, Q_2) solving $H_1^1(Q_1, Q_2) = H_2^1(Q_1, Q_2) = 0$ and any Q_1 solving $H_1^2(Q_1) = 0$ satisfies the property that $Q_1 \in [0, A]$. When $Q_1 \in [0, A]$ and we fix $Q_2 = A - Q_1$, then $Q_2 \in [0, A]$. It follows that $\int_0^{Q_2} (Q_2 - r_2) g(Q_1, r_2) dr_2 + \int_0^{Q_2} \int_{Q_1}^{\infty} [1 - h_1(Q_1)(Q_2 - r_2)] g(r_1, r_2) dr_1 dr_2 \geqslant 0$ becomes

$$\int_0^{A-Q_1} (A - Q_1 - r_2) g(Q_1, r_2) dr_2 + \int_0^{A-Q_1} \int_{Q_1}^{\infty} [1 - h_1(Q_1)(A - Q_1 - r_2)]\, g(r_1, r_2) dr_1 dr_2 \geqslant 0.$$

A similar proof works for the implication in the other direction.

(ii)$\Rightarrow$(i): Define $\Theta(Q_2 \mid Q_1) \equiv \int_0^{Q_2} (Q_2 - r_2) g(Q_1, r_2) dr_2 + \int_0^{Q_2} \int_{Q_1}^{\infty} [1 - h_1(Q_1)(Q_2 - r_2)] g(r_1, r_2) dr_1 dr_2$. Then $\Theta(0 \mid Q_1) = 0$, and $\Theta(Q_2 \mid Q_1) \geqslant 0$ if $\partial\Theta(Q_2 \mid Q_1)/\partial Q_2 \geqslant 0$, that is,

$$\frac{\int_{Q_1}^{\infty} g(r_1, Q_2)dr_1}{\int_{Q_2}^{\infty}\int_{Q_1}^{\infty} g(r_1, r_2)dr_1 dr_2} + h_1(Q_1) \geqslant \frac{\int_{Q_2}^{\infty} g(Q_1, r_2)dr_2}{\int_{Q_2}^{\infty}\int_{Q_1}^{\infty} g(r_1, r_2)dr_1 dr_2}. \tag{2.10}$$

Similarly, we can prove the other direction. From the definition of the hazard gradient, then (2.10) is equivalent to condition (ii).

(iii)⇒(ii): Obviously, a sufficient condition for $h_1(Q_1) + \overline{h}_2(Q_1, Q_2) \geqslant \overline{h}_1(Q_1, Q_2)$ is $h_1(Q_1) \geqslant \overline{h}_1(Q_1, Q_2)$. From Theorem 2.1 of Karia and Deshpande (1999), for any Q_1 and Q_2, $h_1(Q_1) \geqslant \overline{h}_1(Q_1, Q_2) \Leftrightarrow RTI(R_2 \mid R_1)$. So, $RTI(R_1 \mid R_2)$ is a sufficient condition for $h_1(Q_1) + \overline{h}_2(Q_1, Q_2) \geqslant \overline{h}_1(Q_1, Q_2)$. The other direction can be proved similarly.

(vi)⇒(v)⇒(iv)⇒(iii): It follows from the facts that $TP_2(R_1, R_2) \Rightarrow RCSI(R_1, R_2) \Rightarrow RTI(R_i \mid R_{3-i})$ and a bivariate normal distribution with $\rho \geqslant 0$ is TP_2 (Barlow and Proschan 1975). ∎

Lemma 2.4.2 allows us to identify associated distributions, including positively correlated bivariate distributions, that satisfy the unimodality conditions. Associated distributions may arise from the effects of such factors as weather, economy, raw material supply, and government policy on each supplier's capacity. Consider the case of a firm which sources auto parts from Japan through multiple suppliers. The capacity of each supplier is affected by the earthquake on March 11. As a result, we expect their capacities to be "positively" correlated and satisfy the conditions in Lemma 2.4.2. Similarly, the weather has a huge impact on the capacities of agricultural product suppliers. When such suppliers are located in the same geographic area, their capacity distributions are expected to satisfy those conditions as well. Throughout this section, we assume that the unimodality conditions hold.

To facilitate the analysis, we define the *(unit) effective purchase cost from supplier i* expressed as a function of his wholesale price c as

$$C_i = C_i(c) \equiv c + \int_0^{\frac{a-bc}{2}} \left(\frac{a-2r}{b} - c\right) dG_i(r). \tag{2.11}$$

Thus, the effective purchase cost $C_i(c_i)$ from supplier i consists of his wholesale price c_i and the unit imputed cost of his unreliability. The latter cost, termed the unreliability cost of supplier i, kicks in only if he is unable to deliver the optimal ordered quantity $(a - bc_i)/2$ derived for the one-supplier case, and it is equal to the marginal profit that the firm would make if he could deliver one additional unit. With each unit increase in the realized capacity of supplier i, the firm's profit increases by $(a - 2r)/b - c_i$. The expected unit cost of unreliability is easily seen to be the integral term in (2.11). Furthermore, the effective purchase cost from supplier i is increasing and convex in his wholesale price. We can now characterize the firm's optimal order quantities by the following theorem.

Theorem 2.4.3 *The firm's optimal order quantities are* [2]

$$\begin{cases} Q_1^* = (a - bc_1)/2, Q_2^* = 0, & \text{if } c_2 \geqslant C_1, \\ Q_1^* = 0, Q_2^* = (a - bc_2)/2, & \text{if } c_1 \geqslant C_2, \\ Q_1^* = \overline{Q}_1, Q_2^* = \overline{Q}_2, & \text{if } c_1 < C_2 \text{ and } c_2 < C_1 \text{ and } \widehat{Q}_1(c_1) + \widehat{Q}_2(c_2) \leqslant A, \\ Q_1^* = \widehat{Q}_1, Q_2^* = \widehat{Q}_2, & \text{otherwise.} \end{cases}$$

Proof of Theorem 2.4.3. Follows from Lemmas 2.4.4-2.4.7.

Let $\Psi_1(x, y) \equiv (x+y)(a-(x+y))/b$ and $\Psi_2(x, y) \equiv [a^2-(\gamma b)^2]/4b+[\gamma(2(x+y)-a+\gamma b)]/2$.

Lemma 2.4.4 *The unique optimal order quantities (Q_1^*, Q_2^*) satisfy*

$$\begin{cases} Q_1^* = (a - bc_1)/2, Q_2^* = 0, & \text{if } c_2 \geqslant C_1(c_1), \\ Q_1^* = 0, Q_2^* = (a - bc_2)/2, & \text{if } c_1 \geqslant C_2(c_2), \\ Q_1^* = \overline{Q}_1, Q_2^* = \overline{Q}_2, & \text{if } C_2^{-1}(c_1) < c_2 < C_1(c_1) \text{ and } \overline{Q}_1(c_1) + \overline{Q}_2(c_2) \leqslant A, \\ (Q_1^*, Q_2^*) \text{ are in Region II}, & \text{otherwise.} \end{cases} \tag{2.12}$$

Proof of Lemma 2.4.4. Region I: $Q_1 \geqslant 0, Q_2 \geqslant 0, Q_1+Q_2 \leqslant A$. In this region, the second term of (2.5) disappears, and the expected profit function becomes

$$\begin{aligned} \Pi(Q_1, Q_2) \quad = \quad & \int_0^{Q_1}\int_0^{Q_2} \Psi_1(r_1, r_2)g(r_1, r_2)dr_2dr_1 + \int_{Q_1}^{\infty}\int_{Q_2}^{\infty} \Psi_1(Q_1, Q_2)g(r_1, r_2)dr_2dr_1 \\ & + \int_0^{Q_1}\int_{Q_2}^{\infty} \Psi_1(r_1, Q_2)g(r_1, r_2)dr_2dr_1 + \int_0^{Q_2}\int_{Q_1}^{\infty} \Psi_1(Q_1, r_2)g(r_1, r_2)dr_2dr_1 \\ & -c_1\left[\int_0^{Q_1} r_1g_1(r_1)dr_1 + Q_1\overline{G}_1(Q_1)\right] - c_2\left[\int_0^{Q_2} r_2g_2(r_2)dr_2 + Q_2\overline{G}_2(Q_2)\right]. \end{aligned}$$

[2]In the remainder of the chapter, we sometimes write $\widehat{Q}_i$ as $\widehat{Q}_i(c_i)$ for $i = 1, 2$ to emphasize the dependence of $\widehat{Q}_i$ on c_i. Similarly, we write $\overline{Q}_i$ as $\overline{Q}_i(c_i)$, for $i = 1, 2$.

For $i = 1, 2$, let

$$\begin{aligned} H_i^1(Q_1, Q_2) &\equiv b\frac{\partial \Pi(Q_1, Q_2)}{\partial Q_i} \\ &= (a - bc_i - 2(Q_1 + Q_2))\overline{G}_i(Q_i) + 2\int_0^{Q_{3-i}} \int_{Q_i}^{\infty} (Q_{3-i} - r_{3-i}) g(r_1, r_2) dr_i dr_{3-i}. \end{aligned}$$

Next, we analyze the curves $H_i^1(Q_1, Q_2) = 0$ in the (Q_1, Q_2)-plane illustrated in Figure 2.1 plotted for $c_1 > c_2$. Applying the implicit function theorem to $H_1^1(Q_1, Q_2) = 0$ gives

$$\frac{dQ_1}{dQ_2} = \frac{2\int_{Q_2}^{\infty}\int_{Q_1}^{\infty} g(r_1, r_2) dr_1 dr_2}{-2\overline{G}_1(Q_1) - g_1(Q_1)[a - bc_1 - 2(Q_1 + Q_2)] - 2\int_0^{Q_2}(Q_2 - r_2) g(Q_1, r_2) dr_2}.$$

By the unimodality conditions, for any points on the curve $H_1^1(Q_1, Q_2) = 0$, we have $\displaystyle\int_0^{Q_2}(Q_2 - r_2) g(Q_1, r_2) dr_2 + \int_0^{Q_2}\int_{Q_1}^{\infty}[1 - h_1(Q_1)(Q_2 - r_2)] g(r_1, r_2) dr_1 dr_2 \geqslant 0$. By $H_1^1(Q_1, Q_2) = 0$, $\displaystyle\int_0^{Q_2}\int_{Q_1}^{\infty}(Q_2 - r_2) g(r_1, r_2) dr_1 dr_2 = -\frac{1}{2}(a - bc_1 - 2(Q_1 + Q_2))\overline{G}_1(Q_1)$. Thus, $2\overline{G}_1(Q_1) + g_1(Q_1)[a - bc_1 - 2(Q_1 + Q_2)] + 2\displaystyle\int_0^{Q_2}(Q_2 - r_2) g(Q_1, r_2) dr_2 \geqslant 2\int_{Q_2}^{\infty}\int_{Q_1}^{\infty} g(r_1, r_2) dr_1 dr_2$. So, on the curve $H_1^1(Q_1, Q_2) = 0$, $-1 \leqslant dQ_1/dQ_2 < 0$; furthermore, $((a - bc_1)/2, 0)$ and $(0, \widetilde{Q}_2)$ lie on the curve, where $\int_0^{\widetilde{Q}_2}\overline{G}_2(r_2) dr_2 = (a - bc_1)/2 < \widetilde{Q}_2$.

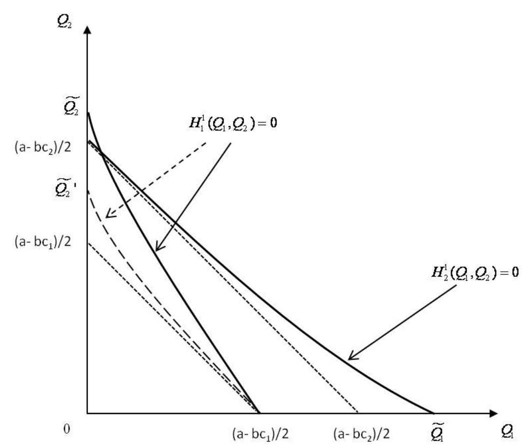


Figure 2.1. Curves $H_1^1(Q_1, Q_2) = 0$ and $H_2^1(Q_1, Q_2) = 0$ if $c_1 > c_2$

By symmetry, on curve $H_2^1(Q_1, Q_2) = 0$, $-1 \leqslant dQ_2/dQ_1 < 0$; furthermore, $((0, (a - bc_2)/2)$ and $(\widetilde{Q}_1, 0)$ lie on the curve, where $\int_0^{\widetilde{Q}_1}\overline{G}_1(r_1) dr_1 = (a - bc_2)/2 < \widetilde{Q}_1$.

If $c_1 > c_2$, then $\widetilde{Q}_1 > (a - bc_2)/2 > (a - bc_1)/2$. Thus, only if $\widetilde{Q}_2 > (a - bc_2)/2$, then there exists a unique interior solution for $H_1^1(Q_1, Q_2) = 0$ and $H_2^1(Q_1, Q_2) = 0$. Otherwise, there is no interior solution. It can be easily verified that $\widetilde{Q}_2 > (a - bc_2)/2$ is equivalent to $c_1 < C_2(c_2)$. So, when $c_2 < c_1 < C_2(c_2)$, the firm's optimal order quantities are $(\overline{Q}_1, \overline{Q}_2)$, the unique interior solution of (2.6). When $c_1 \geqslant C_2(c_2)$, the optimal order quantities are on the boundary. This means that the firm should order from the low-cost supplier alone. When $c_1 \leqslant c_2 < C_1(c_1)$, the proof is similar. Notice that the sum of $\overline{Q}_1$ and $\overline{Q}_2$ may be greater than A. We will show later that if $\overline{Q}_1 + \overline{Q}_2 > A$, then the optimal order quantities are in Region II. In summary, the optimal order quantities are characterized by (2.12).

Next, we check the unimodality of $\Pi(Q_1, Q_2)$. By the FOC and the unimodality conditions,

$$\left.\frac{\partial^2 \Pi(Q_1, Q_2)}{\partial Q_1^2}\right|_{Q_1=\overline{Q}_1, Q_2=\overline{Q}_2}$$
$$= -\frac{2}{b}\left\{\overline{G}_1(\overline{Q}_1) + g_1(\overline{Q}_1)[\frac{a - bc_1}{2} - (\overline{Q}_1 + \overline{Q}_2)] + \int_0^{\overline{Q}_2} (\overline{Q}_2 - r_2) g(\overline{Q}_1, r_2) dr_2\right\} < 0.$$

Similarly, $\left.\frac{\partial^2 \Pi(Q_1, Q_2)}{\partial Q_2^2}\right|_{Q_1=\overline{Q}_1, Q_2=\overline{Q}_2} < 0$. Also,

$$\left.\frac{\partial^2 \Pi(Q_1, Q_2)}{\partial Q_1 \partial Q_2}\right|_{Q_1=\overline{Q}_1, Q_2=\overline{Q}_2} = \left.\frac{\partial^2 \Pi(Q_1, Q_2)}{\partial Q_2 \partial Q_1}\right|_{Q_1=\overline{Q}_1, Q_2=\overline{Q}_2} = -\frac{2}{b}\int_{\overline{Q}_2}^{\infty}\int_{\overline{Q}_1}^{\infty} g(r_1, r_2) dr_1 dr_2 < 0,$$
$$\left.\left\{\frac{\partial^2 \Pi(Q_1, Q_2)}{\partial Q_1^2} \cdot \frac{\partial^2 \Pi(Q_1, Q_2)}{\partial Q_2^2} - \frac{\partial^2 \Pi(Q_1, Q_2)}{\partial Q_1 \partial Q_2} \cdot \frac{\partial^2 \Pi(Q_1, Q_2)}{\partial Q_2 \partial Q_1}\right\}\right|_{Q_1=\overline{Q}_1, Q_2=\overline{Q}_2} > 0.$$

So, if there exists an interior optimal solution, the Hessian at the optimal point is negative definite. Thus, the profit function is jointly unimodal and it is maximized at the optimal point. Note that when the Hessian of a bivariate function is negative definite at all local optimal points, the function is directionally unimodal, which implies joint unimodality. Wang (2008) established the joint unimodality of the profit function with dual sourcing using the structural properties of the profit function. ∎

Lemma 2.4.5 *The unique optimal order quantities satisfy*

$$\begin{cases} Q_1^* = \widehat{Q}_1, Q_2^* = \widehat{Q}_2, & \text{if } \widehat{Q}_1(c_1) + \widehat{Q}_2(c_2) \geqslant A, \\ (Q_1^*, Q_2^*) \text{ are in Region I}, & \text{otherwise}. \end{cases} \tag{2.13}$$

Proof of Lemma 2.4.5. Region II: $Q_1 + Q_2 > A, Q_1 \leqslant A, Q_2 \leqslant A$. The expected profit function

$$\begin{aligned} \Pi(Q_1, Q_2) &= \int_0^{A-Q_2} \int_0^{Q_2} \Psi_1(r_1, r_2) g(r_1, r_2) dr_2 dr_1 + \int_{A-Q_2}^{Q_1} \int_0^{A-r_1} \Psi_1(r_1, r_2) g(r_1, r_2) dr_2 dr_1 \\ &+ \int_{A-Q_2}^{Q_1} \int_{A-r_1}^{Q_2} \Psi_2(r_1, r_2) g(r_1, r_2) dr_2 dr_1 + \int_0^{A-Q_2} \int_{Q_2}^{\infty} \Psi_1(r_1, Q_2) g(r_1, r_2) dr_2 dr_1 \\ &+ \int_{A-Q_2}^{Q_1} \int_{Q_2}^{\infty} \Psi_2(r_1, Q_2) g(r_1, r_2) dr_2 dr_1 + \int_0^{A-Q_1} \int_{Q_1}^{\infty} \Psi_1(Q_1, r_2) g(r_1, r_2) dr_1 dr_2 \\ &+ \int_{A-Q_1}^{Q_2} \int_{Q_1}^{\infty} \Psi_2(Q_1, r_2) g(r_1, r_2) dr_1 dr_2 + \int_{Q_1}^{\infty} \int_{Q_2}^{\infty} \Psi_2(Q_1, Q_2) g(r_1, r_2) dr_2 dr_1 \\ &- c_1 \left[\int_0^{Q_1} r_1 g_1(r_1) dr_1 + Q_1 \overline{G}_1(Q_1) \right] - c_2 \left[\int_0^{Q_2} r_2 g_2(r_2) dr_2 + Q_2 \overline{G}_2(Q_2) \right]. \end{aligned}$$

For $i = 1, 2$, let

$$H_i^2(Q_i) \equiv b \frac{\partial \Pi(Q_1, Q_2)}{\partial Q_i} = \int_{Q_i}^{\infty} \int_0^{A-Q_i} 2(A - (Q_i + r_{3-i})) g(r_1, r_2) dr_{3-i} dr_i - b(c_i - \gamma) \overline{G}_i(Q_i).$$

Let $\widehat{Q}_i$ be the solution of $H_i^2(Q_i) = 0$. Because $\widehat{Q}_i \in (0, A)$, $(\widehat{Q}_1, \widehat{Q}_2)$ are in Region I or II. If $(\widehat{Q}_1, \widehat{Q}_2)$ are in Region II, then they are the optimal order quantities in Region II. Otherwise, the optimal order quantities for the firm will be in Region I, which will be proved later. In summary, the optimal order quantities are characterized by (2.13).

Next, we check the unimodality of $\Pi(Q_1, Q_2)$. By the FOC and the unimodality conditions,

$$\begin{aligned} \left. \frac{\partial^2 \Pi(Q_1, Q_2)}{\partial Q_1^2} \right|_{Q_1 = \widehat{Q}_1, Q_2 = \widehat{Q}_2} &= -\frac{2}{b} \int_0^{A-\widehat{Q}_1} \int_{\widehat{Q}_1}^{\infty} g(r_1, r_2) dr_1 dr_2 + (c_1 - \gamma) g_1(\widehat{Q}_1) \\ &\quad -\frac{1}{b} \int_0^{A-\widehat{Q}_1} (a - \gamma b - 2\widehat{Q}_1 - 2r_2) g(\widehat{Q}_1, r_2) dr_2 < 0, \end{aligned}$$

Similarly, $\frac{\partial^2\Pi(Q_1,Q_2)}{\partial Q_2^2}\Big|_{Q_1=\widehat{Q}_1,Q_2=\widehat{Q}_2} < 0$, $\frac{\partial^2\Pi(Q_1,Q_2)}{\partial Q_1\partial Q_2}\Big|_{Q_1=\widehat{Q}_1,Q_2=\widehat{Q}_2} = \frac{\partial^2\Pi(Q_1,Q_2)}{\partial Q_2\partial Q_1}\Big|_{Q_1=\widehat{Q}_1,Q_2=\widehat{Q}_2} = 0$, and

$$\left\{\frac{\partial^2\Pi(Q_1,Q_2)}{\partial Q_1^2}\cdot\frac{\partial^2\Pi(Q_1,Q_2)}{\partial Q_2^2}-\frac{\partial^2\Pi(Q_1,Q_2)}{\partial Q_1\partial Q_2}\cdot\frac{\partial^2\Pi(Q_1,Q_2)}{\partial Q_2\partial Q_1}\right\}\Bigg|_{Q_1=\widehat{Q}_1,Q_2=\widehat{Q}_2} > 0.$$

Thus, if there exists an interior optimal solution, then the Hessian at the optimal point is negative definite. Thus, the profit function is jointly unimodal and it is maximized at the optimal point. ∎

Lemma 2.4.6 (i) $\overline{Q}_1(c_1)+\overline{Q}_2(c_2) = A \Leftrightarrow \widehat{Q}_1(c_1)+\widehat{Q}_2(c_2) = A$; (ii) $\overline{Q}_1(c_1)+\overline{Q}_2(c_2) > A \Leftrightarrow \widehat{Q}_1(c_1)+\widehat{Q}_2(c_2) > A$; (iii) $\overline{Q}_1(c_1)+\overline{Q}_2(c_2) < A \Leftrightarrow \widehat{Q}_1(c_1)+\widehat{Q}_2(c_2) < A$.

Proof of Lemma 2.4.6. (i) " $\Rightarrow$ ". Suppose $\overline{Q}_1(c_1)+\overline{Q}_2(c_2) = A$. By $H_1^1(\overline{Q}_1,\overline{Q}_2) = 0$, $\int_{\overline{Q}_1}^{\infty}\int_0^{A-\overline{Q}_1}(a-\gamma b-2(\overline{Q}_1+r_2))g(r_1,r_2)dr_2dr_1 - b(c_1-\gamma)\overline{G}_1(\overline{Q}_1) = 0$. By $H_1^2(\widehat{Q}_1) = 0$, $\displaystyle\int_{\widehat{Q}_1}^{\infty}\int_0^{A-\widehat{Q}_1}(a-\gamma b-2(\widehat{Q}_1+r_2))g(r_1,r_2)dr_2dr_1 - b(c_1-\gamma)\overline{G}_1(\widehat{Q}_1) = 0$. Since $\Pi(Q_1,Q_2)$ in Region II is unimodal, $\overline{Q}_1 = \widehat{Q}_1$. Similarly, $\overline{Q}_2 = \widehat{Q}_2$. Consequently, $\widehat{Q}_1(c_1)+\widehat{Q}_2(c_2) = A$.

(ii) " $\Rightarrow$ ". Assume $\overline{Q}_1(c_1)+\overline{Q}_2(c_2) > A$, that is, $\overline{Q}_2 > A-\overline{Q}_1$. Since $H_1^1(\overline{Q}_1,\overline{Q}_2) = 0$, and $\partial H_1^1(Q_1,Q_2)/\partial Q_2 < 0$, we can conclude that $H_1^1(\overline{Q}_1, A-\overline{Q}_1) > 0$. Since $H_1^2(\widehat{Q}_1) = 0$ and $H_1^2(Q_1)$ is unimodal, $\overline{Q}_1 < \widehat{Q}_1$. Similarly, $\overline{Q}_2 < \widehat{Q}_2$. Thus, $\widehat{Q}_1(c_1)+\widehat{Q}_2(c_2) > \overline{Q}_1(c_1)+\overline{Q}_2(c_2) > A$.

(iii) " $\Rightarrow$ ". Interchanging $<$ and $>$ in the last proof gives the result.

(i)(ii)(iii) " $\Leftarrow$ ". From the " $\Rightarrow$ " in (i),(ii), and (iii), we can prove " $\Leftarrow$ " by contradiction. ∎

Lemma 2.4.7 *There exists a unique pair of optimal order quantities for the firm.*

Proof of Lemma 2.4.7. It is completed by proving the following three claims.

Claim 1: *When $c_1 \geqslant C_2(\gamma)$ or $c_2 \geqslant C_1(\gamma)$, the optimal order quantity is in Region I:* Since $\Pi(Q_1,Q_2)$ in Region II is unimodal, to have an interior optimal solution, we must have $H_1^2(0) > 0 \Leftrightarrow c_1 < C_2(\gamma)$. Otherwise, $H_1^2(Q_1) < 0$ for any $Q_1 \in (0, A]$. That is, the optimal solution in Region II is a boundary solution $Q_1 + Q_2 = A$. So the optimal order quantity is in Region I. By symmetry, if $c_2 \geqslant C_1(\gamma)$, the optimal order quantity is in Region I.

Claim 2: *When $c_1 < C_2(\gamma)$, $c_2 < C_1(\gamma)$, and $\widehat{Q}_1 + \widehat{Q}_2 \leqslant A$, the optimal order quantity is in Region I:* If $c_1 < C_2(\gamma)$, then, $H_1^2(0) > 0$. Since $H_1^2(A) = b(\gamma - c_1)\overline{G}_1(A) < 0$, there exists a unique point $\widehat{Q}_1 \in (0, A)$ such that $H_1^2(\widehat{Q}_1) = 0$. By Symmetry, if $c_2 < C_1(\gamma)$, then there exists a unique point $\widehat{Q}_2 \in (0, A)$ such that $H_2^2(\widehat{Q}_2) = 0$. However, if $\widehat{Q}_1 + \widehat{Q}_2 \leqslant A$, then for any point in Region II with $Q_1 > \widehat{Q}_1$, $H_1^2(Q_1) < 0$; and for any point in Region II with $Q_2 > \widehat{Q}_2$, $H_2^2(Q_2) < 0$. These two conditions include all the points in Region II. Thus the optimal solution in Region II is a boundary solution on $Q_1 + Q_2 = A$. So the optimal order quantity is in Region I.

Claim 3: *When $c_1 < C_2(\gamma)$, $c_2 < C_1(\gamma)$, and $\widehat{Q}_1 + \widehat{Q}_2 > A$, the optimal order quantity is in Region II:* By Lemma 2.4.6, when $\widehat{Q}_1 + \widehat{Q}_2 > A$, $\overline{Q}_1 + \overline{Q}_2 > A$. Since the profit function in Region I is unimodal, then for any point in Region I with $Q_1 < \overline{Q}_1$, $H_1^1(Q_1, Q_2) > 0$; and for any point in Region I with $Q_2 < \overline{Q}_2$, $H_2^1(Q_1, Q_2) > 0$. These two conditions include all the points in Region I. Thus, the optimal solution in Region I is a boundary solution $Q_1 + Q_2 = A$. So the optimal order quantity is in Region II. ■

■

Figure 2.2 illustrates Theorem 2.4.3 and demonstrates how the changes in the wholesale prices affect the firm's optimal diversification decision. The feasible area for the wholesale prices is divided into two zones: *Dedication (Single-sourcing) Zone* and *Diversification (Dual-sourcing) Zone*. Dedication Zone consists of two areas: I(1) and I(2), where the firm orders only from supplier 1 or supplier 2, respectively. Diversification Zone, where the firm orders from both suppliers, is subdivided into Diversification Zones II(a) and II(b), where the total order quantity is above or below, respectively, the abundant supply. Moreover, the optimal order quantities in Diversification Zones II(a) and II(b) are characterized by (2.6)

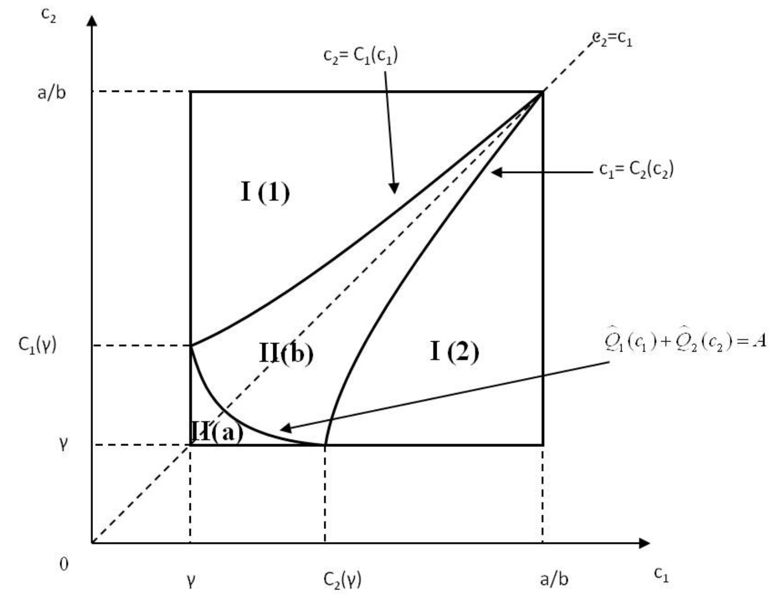


Figure 2.2. Optimal Diversification Decision with Two Dependent Unreliable Suppliers

and (2.7), respectively.

According to Theorem 2.4.3, when a supplier's wholesale price is higher than the effective purchase cost from his rival, the firm should not diversify, and order only from the latter. In other words, even if the supplier with a higher wholesale price is perfectly reliable or his capacity is correlated to his rival's capacity, he may not get any order if his wholesale price is too high. On the other hand, the supplier with a lower wholesale price always get an order from the firm. Therefore, the first part of the insight, i.e., cost is the order qualifier, continues to hold in the two-dependent-supplier case.

Note that Diversification Zone is fully characterized by the marginal distribution of each supplier's capacity and is not affected by their correlation. That is, whether the firm should diversify or not does not depend on the correlation between the two suppliers' capacities. However, the capacity correlation does affect the firm's optimal order quantities. To study the capacity correlation effect, we first present the concept of the supermodular order. A random vector X is said to be greater than another random vector Y in the supermodular order if $E[f(X)] \geqslant E[f(Y)]$ holds for all supermodular functions $f(\cdot)$, provided the expectations exist; see, for example, Shaked and Shanthikumar (2007, Sec. 9.A.4) for more discussions on the supermodular order.

Lemma 2.4.8 (i) *The firm's Diversification Zone is independent of the capacity correlation;* (ii) *In Diversification Zone* II(b)*, the firm's optimal total order quantity decreases as the capacity correlation increases in the sense of the supermodular order.* (iii) *In Diversification Zone* II(a)*, the firm's optimal order quantity for each supplier decreases as the capacity correlation increases in the sense of the supermodular order.*

Proof of Lemma 2.4.8. The proof consists of proving three claims: (i) $C_i(c_i)$ is independent of the capacity correlation; (ii) $\overline{Q}_1 + \overline{Q}_2$ decreases as correlation increases; (iii) $(\widehat{Q}_1, \widehat{Q}_2)$ decreases as correlation increases. The proof of (i) is trivial. To prove (ii), we first analyze how $H_1^1(Q_1, Q_2)$ changes w.r.t. the capacity correlation. Define

$$\phi_1(r_1, r_2) \equiv \begin{cases} Q_2 - r_2, & \text{if } 0 \leqslant r_2 \leqslant Q_2, r_1 \geqslant Q_1, \\ 0\,, & \text{otherwise.} \end{cases}$$

It can be verified that $\phi_1(r_1, r_2)$ is a submodular function. From Shaked and Shanthikumar (2007, 9.A.4, p.395), when the suppliers' capacities increase in the supermodular order, the expectation of any submodular function of (r_1, r_2) decreases. Thus, when the capacity correlation increases in the sense of the supermodular order, $E[\phi_1(r_1, r_2)]$ decreases. Note that $H_1^1(Q_1, Q_2) = (a - bc_1 - 2(Q_1 + Q_2))\overline{G}_1(Q_1) + 2E[\phi_1(r_1, r_2)]$. So, for any fixed Q_1 and Q_2, $H_1^1(Q_1, Q_2)$ decreases as the capacity correlation increases. Since $\partial H_1^1(Q_1, Q_2)/\partial Q_2 < 0$, then for any fixed Q_1, Q_2' obtained from $H_1^1(Q_1, Q_2') = 0$ decreases. This means that the curve $H_1^1(Q_1, Q_2) = 0$ in Figure 2.1 will shift downward. However, the two ending points $(0, \widetilde{Q}_2)$ and $((a - bc_1)/2, 0)$ stay the same, because they are not related to the capacity correlation. Also, from the proof of Lemma 2.4.4, we know that on the curve $H_1^1(Q_1, Q_2) = 0$, $-1 \leqslant dQ_1/dQ_2 < 0$. By symmetry, we can show that when the capacity correlation increases, the curve $H_2^1(Q_1, Q_2) = 0$ in Figure 2.1 shifts downward as well, while the two ending points $(0, (a - bc_2)/2)$ and $(\widetilde{Q}_1, 0)$ stay the same. Also from the proof of Lemma 2.4.4, we know that on the curve $H_2^1(Q_1, Q_2) = 0$, $-1 \leqslant dQ_2/dQ_1 < 0$. Consequently, when the capacity correlation increases, the crossing point of $H_1^1(Q_1, Q_2) = 0$ and $H_2^1(Q_1, Q_2) = 0$, i.e. $(\overline{Q}_1, \overline{Q}_2)$, can only move inside the region bounded by $H_1^1(Q_1, Q_2) = 0$, $H_2^1(Q_1, Q_2) = 0$, and $Q_1 + Q_2 = (a - bc_2)/2$. In this region, the value $Q_1 + Q_2$ of any point (Q_1, Q_2) is less than the original $(\overline{Q}_1 + \overline{Q}_2)$. Thus, $\overline{Q}_1 + \overline{Q}_2$ decreases as the capacity correlation increases.

To prove claim (iii), we first analyze how $H_1^2(Q_1)$ changes w.r.t the capacity correlation. Define

$$\phi_2(r_1, r_2) \equiv \begin{cases} a - b\gamma - 2(Q_1 + r_2), & \text{if } 0 \leqslant r_2 \leqslant A - Q_1, r_1 \geqslant Q_1, \\ 0\,, & \text{otherwise.} \end{cases}$$

It can be verified that $\phi_2(r_1, r_2)$ is a submodular function. Thus, when the capacity correlation increases, $E[\phi_1(r_1, r_2)]$ decreases. Note that $H_1^2(Q_1) = -b(c_1 - \gamma)\overline{G}_1(Q_1) + E[\phi_2(r_1, r_2)]$. So for any fixed Q_1, $H_1^2(Q_1)$ decreases as the capacity correlation increases. By the unimodality conditions, $\Pi(Q_1, Q_2)$ is unimodal in Region II and achieves its maximum at $(\widehat{Q}_1, \widehat{Q}_2)$. Thus, $H_1^2(Q_1) > 0$ if $Q_1 > \widehat{Q}_1$; $H_1^2(Q_1) < 0$ if $Q_1 < \widehat{Q}_1$. So, as the capacity correlation increases, $\widehat{Q}_1$ decreases. Similarly, as the capacity correlation increases, $\widehat{Q}_2$ decreases.

■

In our computational study, we have found cases where one individual order quantity increases as capacity correlation increases in the sense of the supermodular order in Diversification Zone II(b). Lemma 2.4.8 characterizes how the capacity correlation affects the optimal order quantities in each Diversification Zone. However, as the capacity correlation changes, the boundary between Diversification Zone II(a) and II(b) shifts. To draw general conclusions on how capacity correlation affects the optimal order quantities, we next analyze the boundary $\{(c_1, c_2) : \widehat{Q}_1(c_1) + \widehat{Q}_2(c_2) = A\}$ separating Diversification Zones II(a) and II(b). First, we characterize its slope by applying the implicit function theorem:

$$\frac{dc_2}{dc_1} = -\frac{\partial \widehat{Q}_1(c_1)/\partial c_1}{\partial \widehat{Q}_2(c_2)/\partial c_2} = -\frac{\overline{G}_1(\widehat{Q}_1(c_1))A_2}{\overline{G}_2(\widehat{Q}_2(c_2))A_1} \leqslant 0,$$

where for $i = 1, 2$,

$$A_i = 2\int_0^{A-\widehat{Q}_i(c_i)} \int_{\widehat{Q}_i(c_i)}^{\infty} g(r_1, r_2) dr_i dr_{3-i} - b(c_i - \gamma) g_i(\widehat{Q}_i(c_i)) + 2\int_0^{A-\widehat{Q}_i(c_i)} (A - \widehat{Q}_i(c_i) - r_{3-i}) g(\mathbf{y^i}) dr_{3-i} \geqslant 0,$$

with $\mathbf{y^1} = (\widehat{Q}_1(c_1), r_2)$ and $\mathbf{y^2} = (r_1, \widehat{Q}_2(c_2))$.

The nonnegativity of A_i follows from the unimodality conditions. Also, we can identify two ending points of the boundary curve as $(\gamma, C_1(\gamma))$ and $(C_2(\gamma), \gamma)$; see Figure 2.2 for an illustration. Lemma 2.4.9 demonstrates how the boundary curve $\{(c_1, c_2) : \widehat{Q}_1(c_1) + \widehat{Q}_2(c_2) = A\}$ changes with respect to the capacity correlation.

Lemma 2.4.9 *The boundary $\{(c_1, c_2) : \widehat{Q}_1(c_1) + \widehat{Q}_2(c_2) = A\}$ in Diversification Zone shifts towards the point (γ, γ) as the capacity correlation increases in the sense of the supermodular order, while the two ending points $(\gamma, C_1(\gamma))$ and $(C_2(\gamma), \gamma)$ stay the same. When the two suppliers' capacities become perfectly positively correlated, Diversification Zone* II(a) *vanishes and the optimal total order quantity becomes $(a - b\min\{c_1, c_2\})/2$.*

Proof of Lemma 2.4.9. Recall that the two ending points of the boundary curve are not affected by the capacity correlation. To see how the curve changes w.r.t. the capacity correlation, we fix c_1 and check how c_2 changes. From Proposition 2.4.8, when the capacity correlation increases, $\widehat{Q}_1$ decreases. Therefore, $\widehat{Q}_2$ has to increase on the curve $\widehat{Q}_1 + \widehat{Q}_2 = A$. If c_2 is fixed, then $\widehat{Q}_2$ decreases as the capacity correlation increases. When the capacity correlation is fixed, $\widehat{Q}_2$ decreases as c_2 increases. Thus, when the capacity correlation increases, c_2 has to decrease to have an increased $\widehat{Q}_2$. So, when c_1 is fixed, c_2 will decrease as the capacity correlation increases on the curve $\widehat{Q}_1 + \widehat{Q}_2 = A$. Similarly, when c_2 is fixed, c_1 will decrease as the capacity correlation increases on the same curve.

When the two suppliers are perfectly positively correlated, $r_1 = r_2$ with probability 1. Then, in Region II, $\partial\Pi(Q_1, Q_2)/\partial Q_1 = 0$ implies $\widehat{Q}_1 < A/2$. Similarly, $\partial\Pi(Q_1, Q_2)/\partial Q_2 = 0$

implies $\widehat{Q}_2 < A/2$. Thus, $\widehat{Q}_1 + \widehat{Q}_2 < A$. From Lemma 2.4.7, we know that the optimal order quantities are in Region I. Thus, Diversification Zone II vanishes in this case.

To show that the total order quantity satisfies $Q_1^* + Q_2^* = (a - bc_2)/2$ with perfectly correlated capacities, we assume $c_1 > c_2$ without loss of generality. Since the optimal order quantities are in Region I, if the two suppliers are perfectly correlated. In this case, the second term of either $H_1^1(Q_1, Q_2) = 0$ or $H_2^1(Q_1, Q_2) = 0$ becomes zero. It can be proved by contradiction that the only possible case has $H_2^1(Q_1, Q_2) = 0$, and we can see that $Q_1^* + Q_2^* = (a - bc_2)/2$. ∎

From Lemma 2.4.8 and 2.4.9, we obtain the following result regarding how the capacity correlation affects the optimal order quantities.

Theorem 2.4.10 *In Diversification Zone, the firm's optimal total order quantity decreases as the capacity correlation increases in the sense of the supermodular order.*

Proof of Theorem 2.4.10. In Diversification Zone, there are three possible areas where the point (c_1, c_2) representing the suppliers' wholesale prices may reside: (i) in Zone I; (ii) on the curve that separates Zones I and II; (iii) in Zone II. In case (i), by Lemma 2.4.9, when the capacity correlation increases, (c_1, c_2) remains in the same zone. By Lemma 2.4.8, the optimal total order quantity decreases with the capacity correlation in this Zone. In case (ii), by Lemma 2.4.9, when the capacity correlation increases, the curve shifts so that (c_1, c_2) becomes a point in the Zone I and then stay there. The total order quantity on the curve is equal to the abundant supply and the total order quantity in Zone I is smaller than the abundant supply. So, the optimal total order quantity decreases, and it continues to decrease on account of Lemma 2.4.8. In case (iii), by Lemma 2.4.9, when the capacity correlation increases, (c_1, c_2) first becomes a point on the curve and then it becomes a point in Zone I. While in Zone II, the optimal total order quantity decreases with the capacity correlation in view of Lemma 2.4.8. The total order quantity is larger than the abundant supply in Zone II, and becomes equal when on the curve, and becomes less when in Zone I. After the point moves onto the curve, the proof is the same as case (ii). ∎

Recall that the firm's diversification decision is independent of the suppliers' capacity correlation. However, when the firm uses dual sourcing, the suppliers' capacity correlation does affect the firm's optimal order quantities. Theorem 2.4.10 implies that suppliers should try to differentiate from each other to reduce their capacity dependence in order to obtain large orders from the firm. Interestingly, when the suppliers' wholesale prices are relatively high, the order quantity for an individual supplier may increase in the capacity correlation. This somewhat counter-intuitive result can be explained as follows. The delivered products from these two suppliers are substitutes. As the correlation between these two suppliers increases, the benefit of supply diversification decreases. Therefore the firm's total order quantity decreases. However, when the suppliers' wholesale prices differ significantly, as the correlation between these two suppliers increases, the benefit of purchase cost reduction by ordering more from the cheaper supplier dominates the benefit of supply diversification. Consequently, the firm orders more from the cheaper supplier while reducing its order from the more expensive supplier.

2.5 N Suppliers

In this section we consider the N-supplier case. Without loss of generality, let $c_1 \leqslant c_2 \leqslant \cdots \leqslant c_N$. It can be verified that, as in the two-supplier case, the optimal order quantities must fall in the following two regions: Region I $= \{\mathbf{Q} : 0 \leqslant Q_i \leqslant A, Q \leqslant A, \text{for } i = 1, 2, \ldots, N\}$ and Region II $= \{\mathbf{Q} : 0 \leqslant Q_i \leqslant A, Q \geqslant A, \text{for } i = 1, 2, \ldots, N\}$. Intuitively, for any supplier i, the optimal order quantity should never exceed the abundant supply. Suppose the firm orders $Q_i > A$ from supplier i. If supplier i's capacity turns out to be less than the abundant supply, then the firm can get the same profit by ordering the abundant supply from him; if supplier i's capacity turns out to be greater than the abundant supply, then the firm can get more profit by ordering the abundant supply from him since the extra units will be salvaged at $\gamma < c_i$.

In each of these two regions, by using the marginal analysis similar to the two-supplier case, we can derive the firm's marginal revenue and marginal cost from supplier i:

$$\begin{aligned}
\mathrm{MR}_i &= \begin{cases} \dfrac{E[a-2S|R_i \geqslant Q_i]}{b}, & \text{if } \mathbf{Q} \text{ is in Region I,} \\ \dfrac{E[a-2S|R_i \geqslant Q_i \text{ and } S \leqslant A]}{b} + \gamma \cdot Pr\{R_i \geqslant Q_i \text{ and } S > A\}, & \text{if } \mathbf{Q} \text{ is in Region II,} \end{cases} \\
\mathrm{MC}_i &= c_i \overline{G}_i(Q_i).
\end{aligned}$$

If supplier i's random capacity R_i turns out to be less than his order quantity Q_i, then the firm's marginal revenue and cost from this supplier are both zero. On the other hand, conditional on the full delivery of supplier i, the firm's marginal cost from supplier i is c_i, and its marginal revenue from supplier i depends on the realization of all the other suppliers' capacities. In Region I, based on the total delivery quantity, the marginal revenue is $(a-2S)/b$ as in (2.4). In Region II, if the total delivery quantity is smaller than A, then the marginal revenue is $(a-2S)/b$; otherwise, the marginal revenue is γ.

Moreover, the optimal order quantities in these two regions must satisfy the following KKT conditions for $i = 1, \ldots, n$:

$$\mathrm{MR}_i - \mathrm{MC}_i \leqslant 0 \qquad \text{and} \qquad (\mathrm{MR}_i - \mathrm{MC}_i)Q_i^* = 0.$$

Solving the KKT conditions in Regions I and II yields $(\overline{Q}_1, \overline{Q}_2, \ldots, \overline{Q}_N)$ and $(\widehat{Q}_1, \widehat{Q}_2, \ldots, \widehat{Q}_N)$, respectively, and the optimal order quantity for supplier i are

$$Q_i^* = \begin{cases} \overline{Q}_i, & \text{if } \Pi(\overline{\mathbf{Q}}) \geqslant \Pi(\widehat{\mathbf{Q}}), \\ \widehat{Q}_i, & \text{otherwise .} \end{cases} \tag{2.14}$$

However, solving for $\overline{Q}_i$ or $\widehat{Q}_i$ is not easy in general for the N-supplier problem. With two dependent suppliers, the firm can efficiently make the optimal sourcing decisions by relying on the two key insights developed in Section 2.4: First, the firm's diversification decision does not depend on the supplier capacity correlation. Second, cost takes precedence over reliability when the firm selects suppliers. These insights are consistent with the findings in the literature that the decision to order from a particular supplier depends on that supplier's cost and the other suppliers' reliabilities (Dada et al. 2007). These insights, however, are no longer true in the N-supplier case, as shown by the following example.

Example 2.5.1 Consider three suppliers. Assume that the marginal distribution for each supplier's capacity is uniform between 0 and 5. Further, $Pr\{R_1 = R_2\} = 1$ and $Pr\{R_1 + R_3 = 5\} = 1$. That is, R_1 and R_2 are perfectly positively correlated, and R_1 and R_3 are perfectly negatively correlated. Let $a = 10$, $b = 1$, that is, the firm's demand function is $10-p$ with p denoting the retail price. In addition, assume that $\gamma = \delta = 0$. In this case, when $c_1 = 1.0, c_2 = 2.3, c_3 = 2.9$, the optimal order quantities are $Q_1^* = 4, Q_2^* = 0, Q_3^* = 2.1$ with an expected profit of 16.1. If, on the other hand, the firm ignores the capacity dependence among the suppliers and makes its sourcing decision based on the cost-first reliability-second insight, the firm would source from suppliers 1 and 2 with $Q_1 = 3.1, Q_2 = 1.7$. Its expected profit would be 15.29.

As can be seen from the example, other than supplier 1, the firm should choose to order from supplier 3 instead of supplier 2, whose wholesale price is lower than that of supplier 3. Consequently, in the case of N dependent suppliers, the insight that cost takes precedence over reliability does not hold in general. Note that other researchers have obtained similar results for cases with discrete demand distributions (Swaminathan and Shanthikumar 1999) and dynamic pricing (Feng and Shi 2012). From the example we also see that taking into consideration of the supply capacity dependence when making sourcing decisions improves the firm's profit by 5.3%. Therefore, it is important for the firm, when making sourcing decisions, to not ignore the supply capacity dependence.

To solve the N-dependent-supplier problem efficiently, we extend the effective purchase cost concept from one supplier to a group of Consider the firm only orders from supplier Z. Let

$$C_Z = \begin{cases} \dfrac{E[a - 2S_Z^*]}{b}, & \text{if } Q_Z^* \leqslant A, \\ \dfrac{E[a - 2S_Z^*|S_Z^* \leqslant A]}{b} + \gamma \cdot Pr\{S_Z^* > A\}, & \text{otherwise,} \end{cases} \tag{2.15}$$

be the effective purchase cost from supplier Z, where Q_Z^* is the firm's optimal total order quantity for supplier Z and S_Z^* is the total delivery quantity after the firm making the optimal sourcing decisions with supplier Z. The effective purchase cost, essentially, is the price that the firm is willing to pay for an extra unit after the goods are delivered. It can be easily verified

Next, we develop two structural properties of the firm's optimal diversification decision. Define the optimal sourcing set $\mathbb{Z}^* \subseteq \{1, 2, \ldots, N\}$ such that $Q_i^* > 0$ for any $i \in \mathbb{Z}^*$.

Theorem 2.5.1 *Consider a set* $\mathbb{Z} \subseteq \{1, 2, \ldots, N\}$ *and its complement* $\mathbb{Z}^c \equiv \{1, 2, \ldots, N\} \backslash \mathbb{Z}$. *Let* l *be the smallest element and* m *be the largest element in* $\mathbb{Z}^c$. (i) *If* $C_Z \leqslant c_l$, *then any subset of* $\{\mathbb{Z}, l, \ldots, m\}$ *except* $\mathbb{Z}$ *is not the optimal sourcing set;* (ii) *If* $c_l < C_Z \leqslant c_m$, *there must exist* $j, k \in \mathbb{Z}^c$ *such that* $C_Z > c_i$ *for all* $i \in \mathbb{Z}^c$ *with* $i \leqslant j$ *and* $C_Z \leqslant c_i$ *for all* $i \in \mathbb{Z}^c$ *with* $i \geqslant k$. *Moreover, any subset of* $\{\mathbb{Z}, k, \ldots, m\}$ *is not the optimal sourcing set;* (iii) *Otherwise,* $\mathbb{Z}$ *is not the optimal sourcing set.*

Proof of Theorem 2.5.1. (ii) Suppose $c_j < C_Z \leqslant c_k$. This implies that the wholesale price the firm is willing to pay for an extra unit is not larger than c_k. Since Theorem 2.4.3 holds for two dependent suppliers with any continuous capacity distributions, we can regard supplier Z as a supplier with the effective purchase cost C_Z. By Theorem 2.4.3, even if there exists a perfectly reliable supplier with wholesale price c_k, it is not optimal for the firm to order from him. Therefore, it is not optimal for the firm to order from any unreliable supplier whose wholesale price is larger than c_k. Furthermore, for any combination of suppliers, whose wholesale prices are larger than c_k, regarded as one supplier, his equivalent wholesale price will be larger than c_k. To sum up, none of the subsets of $\{\mathbb{Z}, k, \ldots, m\} \backslash \mathbb{Z}$ can be an optimal sourcing set. Proofs of (i) and (iii) follow similarly. ■

Theorem 2.5.1 underlines the importance of the effective purchase cost concept. When a supplier's wholesale price is greater than the firm's effective purchase cost from a group of suppliers, this supplier will not be sourced from. However, one should note that even when a supplier's wholesale price is lower than the effective purchase cost from a group of suppliers, this supplier still may not be sourced from. Interestingly, the following theorem points out that the lowest wholesale price is indeed an order qualifier, i.e., the firm will always order from the supplier with the lowest wholesale price.

Theorem 2.5.2 *The firm will always order from the supplier with the lowest wholesale price.*

Proof of Theorem 2.5.2. Let the firm order from a group of suppliers with indices in $\mathbb{Z} \subseteq \{1, 2, \ldots, N\}$. Since the effective purchase cost from a supplier consists of his wholesale price and the unit imputed cost of his unreliability, the effective purchase cost from any unreliable supplier must be greater than his wholesale price. For a group of suppliers, the equivalent wholesale price must be greater than the lowest wholesale price among the suppliers in the group. Therefore, the effective purchase cost from a group of suppliers must be greater than the lowest wholesale price among the suppliers in the group. That is, $C_Z \geqslant c_i$, where i is the smallest number in set $\mathbb{Z}$. Suppose that $1 \notin \mathbb{Z}$, then $C_Z > c_1$. By Theorem 2.5.1, this group of suppliers with indices in $\mathbb{Z}$ is not the optimal sourcing set. Therefore, $Q_1^* > 0$. ■

To provide an intuitive interpretation of Theorem 2.5.2, we regard the original N suppliers as two suppliers: the lowest-wholesale-price supplier 1 and a surrogate supplier representing all other suppliers. Clearly, the average wholesale price of the surrogate supplier for any set of orders will be greater than supplier 1's wholesale price. It follows from Theorem 2.5.1 that it is optimal for the firm to source from supplier 1. For the two supplier case, this result immediately implies that cost is the order qualifier. Theorem 2.5.2 has important managerial implications for the suppliers. Even though the firm does not necessarily follow the "cost first reliability second" rule when selecting suppliers, the supplier with the lowest wholesale price will always receive an order from the firm.

With these structural properties, we now present an algorithm in Figure 2.3 to solve the firm's problem. The reason for the algorithm to ensure optimality follows directly from Theorem 2.5.1. Let Ω be the *candidate pool*, the set that includes all the possible candidates for the optimal sourcing set denoted as Z^*. Initially, Ω is the set of all (non-null) subsets of $\{1, 2, \ldots, N\}$. Let $\|\cdot\|$ be the cardinality operator of a set. In order to illustrate the algorithm, we apply it to Example 2.5.1.

```
Eliminate all the candidates in Ω that do not have 1 as their element.
n = 1.
While (max_{ℤ∈Ω} ||ℤ|| > n) {
        for all ℤ such that ||ℤ|| = n {
                Compute C_Z;
                If C_Z ⩽ c_l, then Ω = ℤ ∪ Ω\all subsets of{ℤ, l, ..., m};
                If c_l < C_Z ⩽ c_m, then Ω = Ω\all subsets of{ℤ, k, ..., m};
                Else, Ω = Ω\ℤ,
                where l (resp. m) is the smallest (resp. largest) element in Z^c,
                and k is the smallest element in ℤ^c such that C_Z ⩽ c_k;
        }
        n = n+1.
}
ℤ* is the element in Ω with the highest profit.
```

Figure 2.3. An Algorithm for Solving the N Dependent Suppliers Problem.

Example 2.5.2 (Continued from Example 2.5.1)

$\Omega = \{\{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$.

Eliminate all the candidates in Ω *that do not have* 1 *as their element.*

$\Omega = \{\{1\}, \{1,2\}, \{1,3\}, \{1,2,3\}\}$.

$n = 1$.

$\max_{\mathbb{Z}\in\Omega} \|\mathbb{Z}\| = 3 > n$.

$\mathbb{Z} = \{1\}, \mathbb{Z}^c = \{2,3\}$: $C_1 = 5.05 \geqslant c_3 \Rightarrow \Omega = \Omega \backslash \{1\} = \{\{1,2\}, \{1,3\}, \{1,2,3\}\}$.

$n = 1 + 1 = 2$.

$\max_{\mathbb{Z}\in\Omega} \|\mathbb{Z}\| = 3 > n$.

$\mathbb{Z} = \{1,2\}, \mathbb{Z}^c = \{3\}$: $C_{\{1,2\}} = 3.528 > c_3 \Rightarrow \Omega = \Omega \backslash \{1,2\} = \{\{1,3\}, \{1,2,3\}\}$.

$\mathbb{Z} = \{1, 3\}, \mathbb{Z}^c = \{2\}$: $C_{\{1,3\}} = 1.882 < c_2 \Rightarrow \Omega = \Omega \backslash \{1, 2, 3\} = \{\{1, 3\}\}$.

$n = 2 + 1 = 3$.

$\max_{\mathbb{Z}\in\Omega} \|\mathbb{Z}\| = 2 < n$.

$\mathbb{Z}^* = \{1, 3\}$.

The algorithm presented in Figure 2.3 can be streamlined under certain conditions. To develop the streamlined procedure, we present one more notion of multivariate dependence in the bivariate case. Random variables S and T are said to be *positively quadrant dependent* if $P[S \leqslant s, T \leqslant t] \geqslant P[S \leqslant s]P[T \leqslant t]$ for all s and t (Shaked and Shanthikumar 2007, Sec.9.A.1). Note that two associated random variables imply that they are positively quadrant dependent. Theorem 2.5.3 further characterizes the firm's optimal diversification decision.

Theorem 2.5.3 *Assume that all pairs of the N suppliers' capacities are positively quadrant dependent. Then, for any two suppliers i and j with $c_i < c_j$, if $Q_i^* = 0$, then $Q_j^* = 0$.*

Proof of Theorem 2.5.3. We prove it by contradiction. Suppose $Q_j^* > 0$. Since any pair of suppliers from the N suppliers are positively quadrant dependent, then $\int_{Q_j^*}^{\infty} \int_{Q_i^*}^{\infty} g_{ij}(r_i, r_j) dr_i dr_j \geqslant \overline{G}_i(Q_i^*)\overline{G}_j(Q_j^*)$ for any $i \neq j$. It follows that $\frac{E[S_i|R_j \geqslant Q_j^*]}{\overline{G}_j(Q_j^*)} \geqslant E[S_i]$, which implies that $\frac{E[S^T|R_j \geqslant Q_j^*]}{\overline{G}_j(Q_j^*)} \geqslant E[S^T]$ and $E[A - S^T|S^T \leqslant A] \geqslant \frac{E[A-S^T|R_j \geqslant Q_j^* \&\& S^T \leqslant A]}{\overline{G}_j(Q_j^*)}$. If $\Pi(\overline{\mathbf{Q}}) \geqslant \Pi(\widehat{\mathbf{Q}})$, then from the marginal analysis, $\frac{E[a-2S^T|R_j \geqslant Q_j^*]}{b} - c_j\overline{G}_j(Q_j^*) = 0$. Therefore, $\text{MR}_i - \text{MC}_i \mid_{Q_i=0} = \frac{E[a-2S^T]}{b} - c_i > \frac{E[a-2S^T|R_j \geqslant Q_j^*]}{b\overline{G}_j(Q_j^*)} - c_j = 0$. Consequently, $Q_i^* > 0$. If $\Pi(\overline{\mathbf{Q}}) \leqslant \Pi(\widehat{\mathbf{Q}})$, then from the marginal analysis, $\frac{a-2E[S^T|R_j \geqslant Q_j^* \&\& S^T \leqslant A]}{b} + \gamma \cdot Pr\{R_j \geqslant Q_j^* \&\& S^T > A\} - c_j\overline{G}_j(Q_j^*) = 0$. This is equivalent to $\frac{2E[A-S^T|R_j \geqslant Q_j^* \&\& S^T \leqslant A]}{b\overline{G}_j(Q_j^*)} + (\gamma - c_j) = 0$. Therefore, $\text{MR}_i - \text{MC}_i \mid_{Q_i=0} = \frac{2E[A-S^T|S^T \leqslant A]}{b} + (\gamma - c_i) > \frac{2E[A-S^T|R_j \geqslant Q_j^* \&\& S^T \leqslant A]}{b\overline{G}_j(Q_j^*)} + (\gamma - c_j) = 0$. Consequently, $Q_i^* > 0$. ■

Theorem 2.5.3 provides a sufficient condition under which the insight – cost first reliability second – continues to hold. It is important to point out that this condition is more general than the condition that all pairs of the N suppliers' capacities are associated, and certainly, this condition is more general than the condition that all pairs of the N suppliers' capacities are independent. More importantly, from Theorem 2.5.3 and Example 2.5.1, we see that when suppliers' capacities are "positively correlated", the firm can pick suppliers based on the cost-first-reliability-second insight. On the other hand, when the suppliers' capacities are "negatively correlated", the firm must take into consideration the suppliers' capacity correlations when making sourcing decisions. Whether a supplier is selected or not depends on this supplier's wholesale price and the firm's effective purchase cost from the selected suppliers. "Negative" correlation with a selected supplier would make the portfolio of these two suppliers more attractive by reducing the firm's effective purchase cost from these two suppliers, thus potentially alter the insight. However, "positive" correlation does not have this impact.

Together, Theorems 2.5.2 and 2.5.3 have the following implications for a supplier. First, supplier may strive to be the lowest-wholesale-price supplier to guarantee an order from the firm. If that is not possible, then he could improve his chance of being sourced from by ensuring his capacity to be negatively correlated with that of the lowest-wholesale-price supplier.

With Theorem 2.5.3, we present a simple procedure in Figure 2.4 to solve the firm's problem when the sufficient condition in Theorem 2.5.3 holds. In this procedure, the suppliers with the n ($n = 1, 2, \ldots, N$) lowest wholesale prices in the candidate pool are always considered first. Then, the wholesale price of supplier $n + 1$ is compared with $C_{\{1,\ldots,n\}}$ to decide whether the firm should source from supplier $n + 1$.

For n from 1 to N {
    Compute $C_{\{1,\ldots,n\}}$;
    If $c_{n+1} < C_{\{1,\ldots,n\}}$, supplier $n + 1$ is picked;
    Else the optimal ordering quantities are $(Q_1^*, \ldots, Q_n^*, 0, \ldots, 0)$; Exit
}

Figure 2.4. A Simple Procedure for Solving the N Dependent Suppliers Problem.

2.6 Concluding Remarks

We study sourcing and pricing decisions of a firm with unreliable dependent suppliers and a price-dependent demand. Our results have the following managerial implications for firms sourcing from unreliable suppliers: First, the firm should always purchase from the supplier with the lowest wholesale price. Second, the firm must take into consideration supplier capacity correlation in addition to their wholesale prices and reliabilities when picking suppliers. Third, from a supplier's perspective, the most effective weapon for him to guarantee an order from the firm is to become the lowest-price supplier.

Our analysis of the two-supplier case can be extended to scenarios that include multiplicative demand forms, initial inventory, or partially reliable suppliers that are able to guarantee supply quantity up to a certain threshold. The analysis of two-supplier case can also be extended to the case where the firm pays for orders instead of deliveries. Most managerial insights continue to hold in these extensions.

We have made several assumptions in this chapter to keep the analysis tractable. In our analysis, we have assumed that the demand takes a specific form. We expect that our results would carry over to more general demands by adjusting the expressions for the abundant supply, the effective purchase cost, the marginal cost/revenue of purchasing from a supplier, and the unimodality conditions, even though the expressions will be difficult and perhaps more involved. In order to focus on the impact of supply uncertainty on the firm's sourcing decisions with responsive pricing, we have assumed that the demand is deterministic. As a future research topic, it would be of interest to see if the main insights developed in this chapter would hold for stochastic demands. Studying how demand uncertainty affects the firm's sourcing decision with unreliable suppliers and responsive pricing would be interesting. In our model, the supplier's capacity is exogenously determined. That is, it is independent of the order quantity. On some occasions, the suppliers' capacities may be affected by the firm's order quantities. In these cases, an endogenous capacity uncertainty is more appropriate. It would be interesting to extend our analysis to these cases as well.

CHAPTER 3

HOW DOES PRICING POWER AFFECT A FIRM'S SOURCING DECISIONS FROM UNRELIABLE SUPPLIERS?

3.1 Synopsis

In this chapter, we attempt to answer the following questions. First, how do the market parameters affect a firm's optimal sourcing decisions? Second, how does a supplier's reliability influence a firm's optimal sourcing decisions? Finally, how does a firm's pricing power affect its sourcing decisions?

To answer these questions, we study a firm's sourcing problem with two unreliable suppliers. We consider the supplier's unreliability in terms of his random capacity as in, for example, Ciarallo et al. (1994), Feng (2010), and Wang et al. (2010). In addition to the existing definition of a supplier's reliability based on the usual stochastic order (*first-order reliability*), we introduce another definition using the convex order (*second-order reliability*). The first one relies on the "size" of a supplier's random capacity, while the other emphasizes the "variability" of the random capacity. We say that a supplier becomes first-order (second-order) more reliable if his capacity increases (resp., decreases) in the usual stochastic order (resp., the convex order). Suppliers differ from one another in terms of their capacity distributions as well as their wholesale prices.

As in Li et al. (2012a), we use the concept of *effective purchase cost* to characterize a firm's optimal diversification decision. The effective purchase cost from a supplier is defined to be his wholesale price plus the imputed cost of his unreliability. We demonstrate that, regardless of the pricing power, a firm with two possible suppliers always orders from the supplier that has a lower wholesale price. A firm orders from only one supplier if the effective purchase cost from this supplier is lower than his rival's wholesale price. Otherwise, the firm orders from both.

A price-setting firm is more inclined to diversify as the potential market size increases or the price sensitivity (of demand) decreases. Furthermore, its optimal order quantity from each supplier increases in the market size. In the case of diversification with relatively low wholesale prices of both suppliers, the firm's optimal order quantity from each supplier decreases in the price sensitivity. However, when both wholesale prices are relatively high, while the order quantity from the more expensive supplier decreases in the price sensitivity, the order quantity from the cheaper supplier may increase as the price sensitivity increases.

A price-taking firm is more inclined to diversify as the market demand increases. However, the market price and the cost of lost goodwill do not influence the firm's propensity to diversify. Moreover, its optimal order quantity from each supplier increases in the market demand. In the case of diversification with relatively low wholesale prices of both suppliers, the firm's optimal order quantity from each supplier increases in the market price, the cost of lost goodwill, and the salvage value of the product. Interestingly, when both wholesale prices are relatively high, while the order quantity from the more expensive supplier increases in the market price and the cost of lost goodwill, the order quantity from the cheaper supplier decreases as the market price or the cost of lost goodwill increases.

For a price-setting firm, the effective purchase cost from a supplier increases as the supplier becomes less reliable in the sense of both "size" and "variability" of the random capacity. This implies that the firm's *diversification zone*, in the two-dimensional space with the suppliers' wholesale prices as the coordinate axes, widens as either or both of the suppliers become less reliable. In particular, a single-sourcing firm leans toward dual sourcing if its suppliers become less reliable. We should note that this nomenclature does not take into account the amounts ordered from the suppliers, which of course will change as discussed in Section 3.4.5, implying a change in the *extent* of diversification as the suppliers' reliabilities change. The impacts of suppliers' reliabilities for the price-taking firm are different: while a price-taking firm's effective purchase cost from a supplier decreases as the supplier becomes first-order more reliable, the effective purchase cost decreases as the supplier becomes second-order more reliable only when the market demand is low. When the market demand is high, the price-taking firm is more inclined to diversify when the supplier's second-order reliability increases.

Regardless of the pricing power, when a firm orders only from one supplier, the optimal order quantity does not depend on his reliability. When a firm is dual sourcing and the wholesale prices are high, the optimal order quantity from a supplier increases in his first-order reliability and his rival's wholesale price, and decreases in his wholesale price and his rival's first-order reliability. Surprisingly, when the wholesale prices are low, the optimal order quantity from a supplier is only affected by his own wholesale price and his rival's reliability. In this case, a supplier cannot receive a larger order by increasing his first-order reliability, but he can be hurt by his rival doing so.

The impacts of suppliers' second-order reliabilities on the optimal order quantities depend on a firm's pricing power. When a price-setting firm is dual sourcing and the wholesale prices are high, the optimal order quantity from a supplier increases in his second-order reliability and decreases in his rival's second-order reliability; when the wholesale prices are low, the optimal order quantity from a supplier is not affected by his own second-order reliability, but decreases in his rival's second-order reliability. However, certain discretion is necessary when a supplier tries to improve his second-order reliability to win a larger order from a price-taking firm. Unlike the regular phenomenon, we show in section 3.4.5 in detail that under certain conditions, a supplier can win, surprisingly, a larger order from a price-taking firm by *reducing* his second-order reliability.

The remainder of the paper is organized as follows. In section 3.2 we review the related literature. In section 3.3 we introduce the model for both price-setting and price-taking firms with several basic assumptions. In section 3.4, we analyze and solve the problem and then examine the impacts of suppliers' reliabilities on a firm's optimal diversification decisions. We conclude the paper with some discussion of the results in section 3.5.

3.2 Literature Review

This paper is related to two streams of literature. One stream deals with procurement strategies under unreliable supply with exogenously given prices. With one supplier, most studies examine inventory decisions with random yields; see, for example, Yano and Lee (1995) and Grosfeld-Nir and Gerchak (2004) for excellent reviews of the literature. On the other hand, fewer investigate the problems with random supply capacity. Ciarallo et al. (1994) demonstrate that in the presence of capacity uncertainty, a base-stock policy remains optimal. While these paper focus on the procurement/production decisions with one unreliable supplier, our paper examine the firm's diversification and ordering decisions with two unreliable suppliers. The benefits of dual-sourcing in the presence of random yields or random leadtime are well established by Gerchak and Parlar (1990), Ramasesh et al. (1991), Parlar and Wang (1993), and so on. Dual-sourcing problem with one unreliable and one perfectly reliable supply source is also studied recently by, for example, Kazaz (2004), Tomlin (2006), and Tomlin and Snyder (2007). The scenario with multiple unreliable suppliers is investigated in Agrawal and Nahmias (1997), Tomlin and Wang (2005), Babich et al. (2007), Tomlin (2009), and Wang et al. (2010).

Studying the impact of a supplier's reliability on a firm's optimal order quantity is not new. Gerchak and Parlar (1990) demonstrate that in an EOQ setting with random yield, if a firm diversifies, then the ratio of the optimal order quantities Q_1^* and Q_2^* satisfies $Q_1^*/Q_2^* = (\mu_1\sigma_2^2)/(\mu_2\sigma_1^2)$, where the two facilities' wholesale prices are the same and μ_i and σ_i are the mean and the standard deviation of the yield of facility i, respectively. Anupindi and Akella (1993) establish a similar relationship in a single-period version of their Model II when the demand is exponential and yields are normally distributed. Agrawal and Nahmias (1997) study the problem with a deterministic demand and N unreliable suppliers and find the same relationship between the order quantities when the wholesale price from all suppliers is the same. Burke et al. (2009) demonstrate that with a stochastic demand and N unreliable suppliers, if all suppliers have the same wholesale prices, the optimal order quantity for an individual supplier increases (resp., decreases) in the mean (resp., standard deviation) of his reliability and decreases (resp., increases) in the mean (resp., standard deviation) of his rival's reliability.

Different from the above papers, a supplier's random capacity in our model has a general distribution. We use the concepts of stochastic orders to study the impacts of supplier reliability. More importantly, we demonstrate that a firm's optimal order quantity from a supplier is not necessarily monotone in the "variability" of the supplier's capacity. Dada et al. (2007) also study a firm's sourcing decision under general assumptions on supply uncertainty. They define a supplier to become more reliable if his capacity increases in the usual stochastic order and demonstrate that the optimal order for a supplier increases in his reliability and decreases in his rival's reliability. In this paper, we define a more general notion of supplier reliability and demonstrate that the impacts of supplier reliability may be different when a firm has different pricing powers.

With two or more independent unreliable suppliers, it is argued in the literature that cost takes precedence over reliability when it comes to selecting suppliers, and reliability affects the order quantity from a selected supplier. Anupindi and Akella (1993) demonstrate that it is always optimal to order some amount from the least expensive supplier, when the initial inventory is insufficient in a multi-period setting where the supply uncertainty can be either in delivery time or delivery quantity or both. Dada et al. (2007) establish the cost-first-reliability-second insight in a single-period setting with more general assumptions on supply uncertainty, where the firm pays for the delivered quantity. Federgruen and Yang (2009) demonstrate a result similar to Dada et al. (2007) in two versions of the planning model–the service constraint model and the total cost model–when the firm pays for every unit ordered. Swaminathan and Shanthikumar (1999), on the other hand, find that in the case of discrete demand, ordering from the most expensive supplier alone may be optimal. In this paper, we confirm the cost-first-reliability-second insight by studying a firm's supply diversification problem with different pricing powers.

The second line of literature related to our paper focuses on inventory decisions with price-dependent demands. Initially, most of the operations management literature, dealing with pricing in inventory/capacity management, focuses on a single product with perfectly reliable supply. Whitin (1955) and Mills (1959, 1962) were among the first who consider endogenous prices in inventory/capacity models. Comprehensive reviews of the newsvendor-type models with endogenous prices have been written by Porteus (1990) and Petruzzi and Dada (1999). Van Mieghem and Dada (1999) discuss price and production postponement strategies as mechanisms for a firm to manage uncertain demand. Bish and Wang (2004) and Chod and Rudi (2005) extend the work of Van Mieghem and Dada (1999) to two-product cases. In particular, Chod and Rudi (2005) demonstrate that with the additional flexibility gained from responsive pricing, the firm can maximize the benefits of favorable demand conditions and mitigate the effects of poor demand conditions, ultimately profiting from variability.

The first work that addresses joint pricing and inventory decisions in the presence of random yield is done by Li and Zheng (2006). They show that the optimal inventory replenishment is characterized by a threshold value. Feng (2010), on the other hand, investigates dynamic pricing and replenishment decisions in the presence of random capacity, and show that a base stock list price policy fails to achieve optimality even with a deterministic demand. Feng (2010) also examines the value of dynamic pricing under supply uncertainty over static pricing. Tomlin and Wang (2008) study production, pricing, downconversion, and allocation decisions in a two-class, stochastic-demand, stochastic-yield coproduction system. They show that recourse pricing benefits a firm more than either downconversion or recourse allocation do. Tang and Yin (2007), who also study responsive pricing under supply uncertainty, demonstrate that a firm can gain a higher expected profit under a responsive pricing policy and examine the impact of yield distribution and system parameters on the optimal order quantities via a numerical analysis. Our work is different from their previous works in that we consider two unreliable suppliers and analytically examine how the influence of a supplier's reliability on a firm's sourcing decision is affected by its pricing power.

Li et al. (2012a) consider a firm's supply diversification problem in the presence of random supply capacity with responsive pricing. They demonstrate that the cost-first-reliability-second insight does not hold when there are more than two correlated suppliers and they focus on the impact of supplier capacity correlation on the firm's optimal diversification decisions. In this paper, by investigating the diversification problems for both price-setting and price-taking firms, we focus on the effects of market parameters and suppliers' reliabilities on their diversification decisions when they have different pricing powers.

3.3 Models

Consider a profit-maximizing firm that may order goods from two suppliers and sells them in the retail market in a single selling season. The suppliers differ from one another in terms of their wholesale prices and reliabilities. They can be either perfectly reliable or unreliable, where the distinction is viewed from the firm's perspective. A supplier is perfectly reliable if he can meet fully the firm's order regardless of the order size; he is unreliable if he cannot. Supplier i ($i = 1, 2$), when unreliable, has a given random production capacity R_i; if supplier i is perfectly reliable, then we have the special deterministic case with $R_i \equiv \infty$. We assume that R_i has the cumulative distribution function (cdf) $G_i(r) \equiv 1 - \overline{G}_i(r) > 0$ and the probability density function (pdf) $g_i(r) \geqslant 0$ for $r > 0$. We also assume that the random variables R_1 and R_2 are independent.

For a price-setting firm, the selling season consists of two stages. In the first stage, the firm orders a quantity Q_i from supplier i at the wholesale price of c_i and receives the quantity $S_i(Q_i) = \min\{Q_i, R_i\}$, $i = 1, 2$. The firm pays a supplier only for the quantity delivered. In the second stage, based on the total received quantity $S(Q_1, Q_2) = S_1(Q_1) + S_2(Q_2)$, the firm decides the unit retail price p for the product. We assume the demand to be deterministic and price-dependent in the additive form, that is, $D(p) = a - bp$, where $a > 0$ is the potential market size and $b > 0$ is the price sensitivity of the demand. To ensure that the firm is able to make a positive profit and avoid trivial cases, we assume that $c_i < a/b$.

We assume holdback rather than clearance, and thus, there could be unsold units, which the firm salvages in a secondary market at a unit price γ. Since the firm pays for the delivery quantity, if either c_1 or c_2 is less than γ, the firm will order infinite amount from the supplier whose wholesale price is less than γ. To exclude this case, we assume that $\gamma < c_i$. The cost of lost goodwill is δ for each unit of the unfulfilled demand.

We use superscript S to indicate the case of the price-setting firm. The price-setting firm's objective is to choose the order quantities $\{Q_1^S, Q_2^S\}$ in the first stage and the retail price p^S in the second stage to maximize its expected profit $\Pi^S\left(Q_1^S, Q_2^S\right)$, which is equal to its expected second-stage profit $E[\Pi_2^S\left(Q_1^S, Q_2^S\right)]$ less its expected purchase cost in the first stage. The firm's problem is:

$$\begin{aligned}
\max_{Q_1^S, Q_2^S \geqslant 0} \Bigg\{ \Pi^S\left(Q_1^S, Q_2^S\right) &= E\left[\Pi_2^S\left(Q_1^S, Q_2^S\right) - \sum_{i=1}^{2} c_i S_i(Q_i^S)\right]\Bigg\}, && (3.1)\\
\text{where} \quad \Pi_2^S\left(Q_1^S, Q_2^S\right) &= \max_{p^S \geqslant 0} \pi(p^S) = \max_{p^S \geqslant 0} \Big\{ p^S \cdot \min\{D(p^S), S(Q_1^S, Q_2^S)\} \\
&\quad + \gamma \cdot \left(S(Q_1^S, Q_2^S) - D(p^S)\right)^+ - \delta \cdot \left(D(p^S) - S(Q_1^S, Q_2^S)\right)^+ \Big\}. && (3.2)
\end{aligned}$$

For the price-taking firm, the selling season only has the first stage that is described earlier. The retail price $\overline{p}$ is given. To avoid trivial cases, we assume that $\overline{p} > c_i$. The deterministic market demand $\overline{D}$ is assumed to be exogenously given. We use superscript T to indicate the case of the price-taking firm. Its objective is to choose the order quantities $\{Q_1^T, Q_2^T\}$ to maximize its expected profit $\Pi^T\left(Q_1^T, Q_2^T\right)$:

$$\begin{aligned}
\max_{Q_1^T, Q_2^T \geqslant 0} \Bigg\{ \Pi^T\left(Q_1^T, Q_2^T\right) &= E\Bigg[\overline{p} \cdot \min\{\overline{D}, S(Q_1^T, Q_2^T)\} + \gamma \cdot \left(S(Q_1^T, Q_2^T) - \overline{D}\right)^+ \\
&\quad - \delta \cdot \left(\overline{D} - S(Q_1^T, Q_2^T)\right)^+ - \sum_{i=1}^{2} c_i S_i(Q_i^T)\Bigg]\Bigg\}. && (3.3)
\end{aligned}$$

3.4 Analysis

3.4.1 The Price-Setting Firm

For the price-setting firm's problem, we specialize the results from Li et al. (2012) to the case of two independent suppliers. The abundant supply $A \equiv (a - b\gamma)/2$ in Li et al. (2012) refers to the threshold delivery quantity, above which the firm's marginal revenue becomes the same as the salvage value γ. We also define the (unit) effective purchase cost from supplier i expressed as a function of his wholesale price c as

$$C_i^S(c) \equiv c + \int_0^{\frac{a-bc}{2}} \left(\frac{a-2r}{b} - c \right) dG_i(r). \tag{3.4}$$

Theorem 2 in Li et al. (2012) applied to the case of two independent suppliers characterizes the firm's optimal order quantities from the suppliers as given below.

Corollary 3.4.1 (of Theorem 2 in Li et al. (2012)) *The price-setting firm's optimal order quantities are*

$$\begin{cases} Q_1^{S*} = (a - bc_1)/2, Q_2^{S*} = 0, & \textit{if } c_2 \geqslant C_1^S(c_1), \\ Q_1^{S*} = 0, Q_2^{S*} = (a - bc_2)/2, & \textit{if } c_1 \geqslant C_2^S(c_2), \\ Q_1^{S*} = \overline{Q}_1^S, Q_2^{S*} = \overline{Q}_2^S, & \textit{if } c_1 < C_2^S(c_2) \textit{ and } c_2 < C_1^S(c_1) \textit{ and } \widehat{Q}_1^S(c_1) + \widehat{Q}_2^S(c_2) \leqslant A, \\ Q_1^{S*} = \widehat{Q}_1^S, Q_2^{S*} = \widehat{Q}_2^S, & \textit{otherwise}, \end{cases}$$

where, for $i = 1, 2$, $\overline{Q}_i^S$ is the solution of

$$a - bc_i - 2Q_i - 2\int_0^{\frac{a-bc_{3-i}}{2} - \int_0^{Q_i} \overline{G}_i(r)dr} \overline{G}_{3-i}(r)dr = 0, \tag{3.5}$$

and $\widehat{Q}_i^S$ is the solution of

$$2\int_0^{A-Q_i} G_{3-i}(r)dr - b(c_i - \gamma) = 0. \tag{3.6}$$

Proof of Corollary 3.4.1. The proof is referred to Li et al. (2012). ∎

Figure 3.1(a) illustrates Corollary 3.4.1 and demonstrates how the changes in the suppliers' wholesale prices affect the price-setting firm's optimal diversification decision. The feasible area for the wholesale prices is divided into two zones: the dedication zone and the diversification zone. The dedication zone consists of two areas: I(1) and I(2), where the firm orders only from supplier 1 or supplier 2, respectively. The diversification zone, where the firm orders from both suppliers, is subdivided into diversification zones I and II, where the total order quantity is less or greater, respectively, than the abundant supply. Moreover, the optimal order quantities in diversification zones I and II are characterized by (3.5) and (3.6), respectively. Next, we study the impacts of market conditions on the firm's diversification decision.

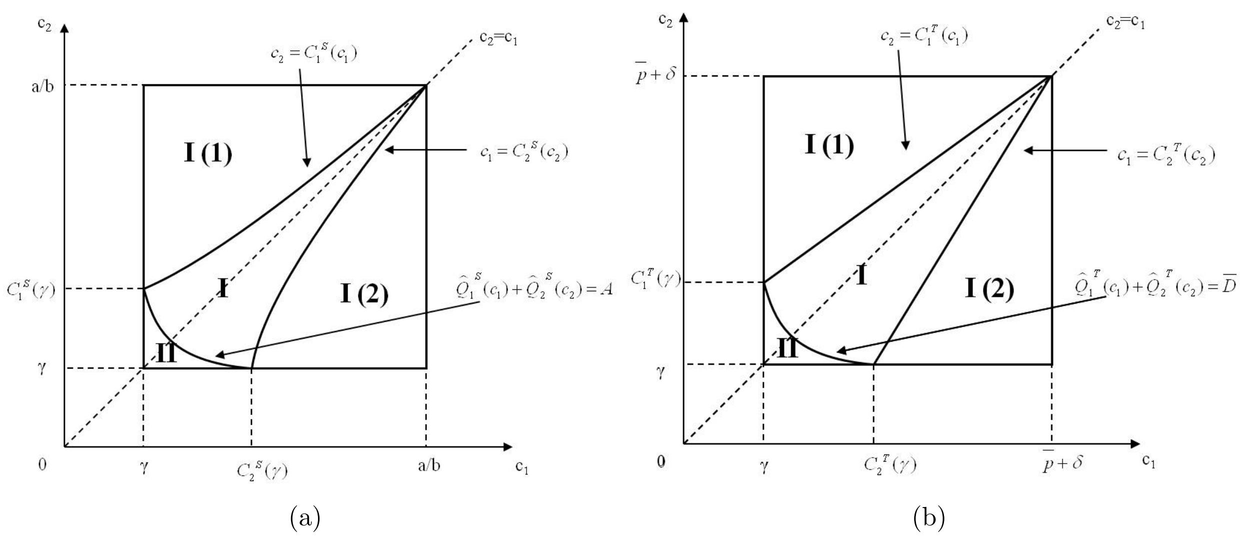


Figure 3.1. Optimal Diversification Decision for (a) the Price-Setting Firm and (b) the Price-Taking Firm

Proposition 3.4.2 *The effective purchase cost $C_i^S(c_i)$ increases in the potential market size and decreases in the price sensitivity.*

Proof of Proposition 3.4.2. It can be easily verified that $\partial C_i^S(c_i)/\partial a = G_i((a - bc_i)/2)/b > 0$ and $\partial C_i^S(c_i)/\partial b = -[bc_i G_i((a-bc_i)/2) + 2\int_0^{\frac{a-bc_i}{2}} G_i(r)dr]/b^2 < 0$. ■

When the potential market size (resp., the price sensitivity) increases (resp., decreases), the optimal order quantity from a sole supplier increases, and thus the probability for a supplier to deliver the optimal quantity decreases. Meanwhile, the firm's marginal revenue increases, which leads to an increase of the opportunity cost of an undelivered unit. Therefore, the imputed cost of a supplier's unreliability increases, which in turn increases the effective purchase cost from him. By Corollary 3.4.1, Proposition 3.4.2 implies that as the potential market size (resp., the price sensitivity) increases (resp., decreases), the diversification zone becomes larger. However, note that when the potential market size (resp., the price sensitivity) increases (resp., decreases), the feasible area for the wholesale prices and the dedication zone become larger as well. Then how does the area of the diversification zone as a fraction of the feasible area change with respect to the potential market size and the price sensitivity? We have the following result.

Proposition 3.4.3 *The area of the diversification zone as a fraction of the feasible area increases in the potential market size and decreases in the price sensitivity.*

Proof of Proposition 3.4.3. Consider the diversification zone above $c_2 = c_1$. For any fixed value of c_1, it is sufficient to show that (the distance between the curves $c_2 = C_1^S(c_1)$ and $c_2 = c_1$)/(the distance between the curves $c_2 = a/b$ and $c_2 = c_1$), i.e. $\left(\int_0^{\frac{a-bc_1}{2}} \left(\frac{a-2r}{b} - c\right) dG_1(r)\right) / \left(a/b - c_1\right)$ increases in a and decreases in b, which can be verified. The proof is similar for the diversification zone below $c_2 = c_1$. ■

For a price-dependent demand $D(p) = a - bp$, when the potential market size (resp., the price sensitivity) increases (resp., decreases), the absolute value of the price elasticity of the demand decreases. Therefore, the impact of a firm's ability to adjust the retail price on the demand is reduced. To compensate for this pricing power reduction and mitigate the impact of supply uncertainty, the firm's propensity to diversify naturally increases.

Next, we characterize the impacts of the market conditions on the optimal order quantities.

Proposition 3.4.4 (i) *The optimal order quantity from each supplier increases in the potential market size.* (ii) *In the dedication zone and diversification zone* II*, the optimal order quantity from each supplier decreases in the price sensitivity.* (iii) *In diversification zone* I*, the total order quantity decreases in the price sensitivity.* (iv) *In diversification zone* I*, the optimal order quantity from the more expensive (in the sense of wholesale price) supplier decreases in the price sensitivity.* (v) *In diversification zone* I*, the optimal order quantity from the less expensive (in the sense of wholesale price) supplier (say, supplier i) increases in the price sensitivity when $c_i < c_{3-i}\overline{G}_{3-i}(\overline{Q}_{3-i}^S)$.*

Proof of Proposition 3.4.4. (i) In dedication zone, $\partial Q^{S*}/\partial a = 1/2 > 0$. In diversification zone I, for $i = 1, 2$,

$$\frac{\partial \overline{Q}_i^S}{\partial a} = \frac{G_{3-i}(\frac{a-bc_{3-i}}{2} - \int_0^{\overline{Q}_i^S} \overline{G}_i(r)dr)}{2 - 2\overline{G}_i(\overline{Q}_i^S)\overline{G}_{3-i}(\frac{a-bc_{3-i}}{2} - \int_0^{\overline{Q}_i^S} \overline{G}_i(r)dr)} > 0.$$

In diversification zone II, for $i = 1, 2$, $\partial \widehat{Q}_i^S/\partial a = 1/2 > 0$.

(ii) In the dedication zone, $\partial Q^{S*}/\partial b = -c/2 < 0$. In diversification zone II, for $i = 1, 2$,

$$\frac{\partial \widehat{Q}_i^S}{\partial b} = \frac{-(c_i - \gamma) - \gamma G_{3-i}(A - \widehat{Q}_i^S)}{2G_{3-i}(A - \widehat{Q}_i^S)} < 0.$$

(iii) Without loss of generality, we assume $c_1 \geqslant c_2$. In diversification zone I, $\overline{Q}_1^S + \overline{Q}_2^S = (a - bc_2)/2 + \int_0^{\overline{Q}_1^S} G_1(r_1)dr_1$. By claim (iv), as b increases, $\overline{Q}_1^S$ decreases. Thus, both terms on the RHS of previous equation decrease. Consequently, the firm's optimal total order quantity decreases.

(iv) In the diversification zone I, for $i = 1, 2$,

$$\frac{\partial \overline{Q}_i^S}{\partial b} = \frac{-c_i + c_{3-i}\overline{G}_{3-i}(\frac{a-bc_{3-i}}{2} - \int_0^{\overline{Q}_i^S} \overline{G}_i(r)dr)}{2 - 2\overline{G}_i(\overline{Q}_i^S)\overline{G}_{3-i}(\frac{a-bc_{3-i}}{2} - \int_0^{\overline{Q}_i^S} \overline{G}_i(r)dr)}.$$

If $c_i \geqslant c_{3-i}$, then $\partial \overline{Q}_i^S/\partial b < 0$.

(v) From (iv), it is straightforward that, if $c_i < c_{3-i}\overline{G}_{3-i}(\overline{Q}_{3-i}^S)$, then $\partial \overline{Q}_i^S/\partial b > 0$. ■

When the potential market size increases, the firm's marginal revenue of ordering from either supplier increases and, therefore, the firm orders more from each supplier. In the dedication zone and diversification zone II, the firm's marginal revenue from a supplier does not depend on its order quantity from the other supplier. Therefore, when the price sensitivity increases, the firm's marginal revenue of ordering from each supplier decreases; consequently, the firm orders less from each supplier. In diversification zone I, the firm's marginal revenue from a supplier depends on its order quantities from both suppliers. When the two suppliers' wholesale prices are close, as the price sensitivity increases, the firm's marginal revenue decreases from both suppliers. To compensate for this loss, the firm has to reduce its order quantities from both suppliers. Interestingly, when one supplier's wholesale price is sufficiently low, the optimal order quantity for this supplier *increases* when the price sensitivity increases. The intuitive explanation is as follows. The two suppliers are substitutes for the firm. When the price sensitivity increases, the firm's marginal revenue decreases. To compensate for this loss, the firm has to reduce its order quantity from its suppliers. When a supplier's wholesale price is sufficiently low, the compensation for the firm's marginal revenue loss by reducing the order quantity from the more expensive supplier is much more significant than by reducing the less expensive supplier's order quantity. Thus, in this case, the firm would rather reduce the order quantity from the more expensive supplier. Once the firm regains the marginal revenue from the more expensive supplier, the firm's marginal revenue is higher than its marginal cost from the less expensive supplier. Consequently, the order quantity for the less expensive supplier increases. Consider a special case: $c_1 < c_2$ and supplier 2 is perfectly reliable. In this case, the firm's order quantity from supplier 1 is increasing in the price sensitivity. This result reveals another benefit of being a lower cost supplier, that is, an increased price sensitivity may bring a larger order and hurt the other supplier.

3.4.2 The Price-Taking Firm

For the price-taking firm, the abundant supply becomes the market demand $\overline{D}$. It can be easily shown that in the single-supplier case, the firm's optimal order quantity is always $\overline{D}$. In this case, the (unit) effective purchase cost from supplier i is modified as

$$C_i^T(c) \equiv c + (\overline{p} + \delta - c)G_i(\overline{D}). \tag{3.7}$$

The effective purchase cost $C_i^T(c_i)$ from supplier i consists of his wholesale price c_i and the unit imputed cost of his unreliability. The latter cost, termed the unreliability cost of supplier i, kicks in only if he is unable to deliver the optimal ordered quantity $\overline{D}$ derived for the one-supplier case, and it is equal to the marginal profit that the firm would make if he could deliver one additional unit. With each unit increase in the realized capacity of supplier i, the firm's profit increases by $\overline{p} + \delta - c_i$. The expected unit cost of unreliability is easily seen to be the second term in (3.7). We can now characterize the firm's optimal sourcing decisions as follows.

Proposition 3.4.5 *The price-taking firm's optimal order quantities are*

$$\begin{cases} Q_1^{T*} = \overline{D}, Q_2^{T*} = 0, & \text{if } c_2 \geqslant C_1^T(c_1), \\ Q_1^{T*} = 0, Q_2^{T*} = \overline{D}, & \text{if } c_1 \geqslant C_2^T(c_2), \\ Q_1^{T*} = \overline{Q}_1^T, Q_2^{T*} = \overline{Q}_2^T, & \text{if } c_1 < C_2^T(c_2),\ c_2 < C_1^T(c_1), \text{ and} \\ & G_1^{-1}\left(\dfrac{c_2-\gamma}{\overline{p}+\delta-\gamma}\right) + G_2^{-1}\left(\dfrac{c_1-\gamma}{\overline{p}+\delta-\gamma}\right) \geqslant \overline{D}, \\ Q_1^{T*} = \widehat{Q}_1^T, Q_2^{T*} = \widehat{Q}_2^T, & \text{otherwise}, \end{cases}$$

where, $\overline{Q}_1^T$ and $\overline{Q}_2^T$ are the solutions of the two simultaneous equations

$$\begin{cases} \dfrac{\overline{G}_1(Q_1)}{\overline{G}_2(\overline{D}-Q_1)} = \dfrac{\overline{p}+\delta-c_2}{\overline{p}+\delta-c_1}, \\ Q_1 + Q_2 = \overline{D}, \end{cases} \tag{3.8}$$

and $\widehat{Q}_1^T$ and $\widehat{Q}_2^T$ are the solutions of

$$\overline{G}_{3-i}(\overline{D}-Q_i) = \frac{\overline{p}+\delta-c_i}{\overline{p}+\delta-\gamma}, \qquad i = 1, 2. \tag{3.9}$$

Proof of Proposition 3.4.5. For the ease of expression, the superscript T is removed from this proof. To solve the problem, we divide the feasible space of $\{Q_1, Q_2\}$ into five regions depending on the specific forms of the expected profit function (3.3) in these regions: Region I: $Q_1 \geqslant 0, Q_2 \geqslant 0, Q_1 + Q_2 \leqslant \overline{D}$; Region II: $Q_1 + Q_2 > \overline{D}, Q_1 \leqslant \overline{D}, Q_2 \leqslant \overline{D}$; Region III: $0 \leqslant Q_1 \leqslant \overline{D}, Q_2 > \overline{D}$; Region IV: $0 \leqslant Q_2 \leqslant \overline{D}, Q_1 > \overline{D}$; Region V: $Q_1 > \overline{D}, Q_2 > \overline{D}$.

Lemma 3.4.6 (i) *The optimal solution in Region* III *must satisfy* $Q_2 = \overline{D}$. (ii) *The optimal solution in Region* IV *must satisfy* $Q_1 = \overline{D}$. (iii) *The optimal solution in Region* V *must satisfy* $Q_1 = Q_2 = \overline{D}$.

Proof of Lemma 3.4.6. Suppose the firm orders $Q_i > \overline{D}$ from supplier i. If supplier i's capacity turns out to be less than $\overline{D}$, then the firm can get the same profit by ordering $\overline{D}$ from him; if supplier i's capacity turns out to be greater than $\overline{D}$, then the firm can get more profit by ordering $\overline{D}$ from him since the extra units will be salvaged at $\gamma < c_i$. ■

Lemma 3.4.7 *The unique optimal order quantities* (Q_1^*, Q_2^*) *satisfy*

$$\begin{cases} Q_1^* = (a - bc_1)/2, Q_2^* = 0, & \text{if } c_2 \geqslant C_1(c_1), \\ Q_1^* = 0, Q_2^* = (a - bc_2)/2, & \text{if } c_1 \geqslant C_2(c_2), \\ Q_1^* = \overline{Q}_1, Q_2^* = \overline{Q}_2, & \text{if } c_1 < C_2(c_2),\ c_2 < C_1(c_1), \text{ and } \widehat{Q}_1(c_1) + \widehat{Q}_2(c_2) \leqslant \overline{D}, \\ (Q_1^*, Q_2^*) \text{ are in Region II}, & \text{otherwise.} \end{cases} \tag{3.10}$$

Proof of Lemma 3.4.7. Let $\Psi_1(x, y) \equiv (x+y)\overline{p} - \delta(\overline{D} - x - y)$ and $\Psi_2(x, y) \equiv D\overline{p} + \gamma(x + y - \overline{D})$. In Region I: $Q_1 \geqslant 0, Q_2 \geqslant 0, Q_1 + Q_2 \leqslant \overline{D}$, the second term of (3.3) disappears, and the expected profit function becomes

$$\begin{aligned} \Pi(Q_1, Q_2) \;=\; & \int_0^{Q_1}\int_0^{Q_2} \Psi_1(r_1, r_2) g_1(r_1) g_2(r_2) dr_2 dr_1 + \int_{Q_1}^{\infty}\int_{Q_2}^{\infty} \Psi_1(Q_1, Q_2) g_1(r_1) g_2(r_2) dr_2 dr_1 \\ & + \int_0^{Q_1}\int_{Q_2}^{\infty} \Psi_1(r_1, Q_2) g_1(r_1) g_2(r_2) dr_2 dr_1 + \int_0^{Q_2}\int_{Q_1}^{\infty} \Psi_1(Q_1, r_2) g_1(r_1) g_2(r_2) dr_2 dr_1 \\ & - c_1\left[\int_0^{Q_1} r_1 g_1(r_1) dr_1 + Q_1\overline{G}_1(Q_1)\right] - c_2\left[\int_0^{Q_2} r_2 g_2(r_2) dr_2 + Q_2\overline{G}_2(Q_2)\right]. \end{aligned}$$

Since for $i = 1, 2$, $\partial\Pi(Q_1, Q_2)/\partial Q_i = (\overline{p} + \delta - c_i)\overline{G}_i(Q_i) > 0$, the optimal solution in Region I must be on the boundary $Q_1 + Q_2 = \overline{D}$. Replace Q_2 with $\overline{D} - Q_1$ in the profit function and take the first order derivative with respect to Q_1 yields the following necessary condition for optimal Q_1:

$$\frac{d\Pi(Q_1)}{dQ_1} = (\overline{p} + \delta - c_1)\overline{G}_1(Q_1) - (\overline{p} + \delta - c_2)\overline{G}_2(\overline{D} - Q_1) \equiv H(Q_1) = 0.$$

Consequently, $d\Pi^2(Q_1)/dQ_1^2 = H'(Q_1) < 0$. It can be verified that if $c_1 \geqslant C_2^T(c_2)$, then $H(Q_1) < 0$ for $Q_1 \in [0, \overline{D}]$. Thus, the firm should order from supplier 2 alone with order quantity $\overline{D}$. On the other hand, if $c_2 \geqslant C_1^T(c_1)$, then $H(Q_1) > 0$ for $Q_1 \in [0, \overline{D}]$. Thus, the firm should order from supplier 1 alone with order quantity $\overline{D}$. As we shall show later, if $\widehat{Q}_1(c_1) + \widehat{Q}_2(c_2) \leqslant \overline{D}$, then the optimal order quantity is on the boundary $Q_1 + Q_2 = \overline{D}$. In summary, the firm's optimal order quantity is characterized by (3.10). ∎

Lemma 3.4.8 *The unique optimal order quantities satisfy*

$$\begin{cases} Q_1^* = \widehat{Q}_1, Q_2^* = \widehat{Q}_2, & \textit{if } \widehat{Q}_1(c_1) + \widehat{Q}_2(c_2) > \overline{D}, \\ (Q_1^*, Q_2^*) \textit{ in on the boundary } Q_1 + Q_2 = \overline{D}, & \textit{otherwise.} \end{cases} \tag{3.11}$$

Proof of Lemma 3.4.8. In Region II: $Q_1 + Q_2 > \overline{D}, Q_1 \leqslant \overline{D}, Q_2 \leqslant \overline{D}$, the expected profit function becomes

$$\begin{aligned}
&\Pi(Q_1, Q_2) = \\
&\int_0^{\overline{D}-Q_2}\int_0^{Q_2} \Psi_1(r_1, r_2)g_1(r_1)g_2(r_2)dr_2dr_1 + \int_{\overline{D}-Q_2}^{Q_1}\int_0^{\overline{D}-r_1} \Psi_1(r_1, r_2)g_1(r_1)g_2(r_2)dr_2dr_1 \\
&+ \int_{\overline{D}-Q_2}^{Q_1}\int_{\overline{D}-r_1}^{Q_2} \Psi_2(r_1, r_2)g_1(r_1)g_2(r_2)dr_2dr_1 + \int_0^{\overline{D}-Q_2}\int_{Q_2}^{\infty} \Psi_1(r_1, Q_2)g_1(r_1)g_2(r_2)dr_2dr_1 \\
&+ \int_{\overline{D}-Q_2}^{Q_1}\int_{Q_2}^{\infty} \Psi_2(r_1, Q_2)g_1(r_1)g_2(r_2)dr_2dr_1 + \int_0^{\overline{D}-Q_1}\int_{Q_1}^{\infty} \Psi_1(Q_1, r_2)g_1(r_1)g_2(r_2)dr_1dr_2 \\
&+ \int_{\overline{D}-Q_1}^{Q_2}\int_{Q_1}^{\infty} \Psi_2(Q_1, r_2)g_1(r_1)g_2(r_2)dr_1dr_2 + \int_{Q_1}^{\infty}\int_{Q_2}^{\infty} \Psi_2(Q_1, Q_2)g_1(r_1)g_2(r_2)dr_2dr_1 \\
&-c_1\left[\int_0^{Q_1} r_1g_1(r_1)dr_1 + Q_1\overline{G}_1(Q_1)\right] - c_2\left[\int_0^{Q_2} r_2g_2(r_2)dr_2 + Q_2\overline{G}_2(Q_2)\right].
\end{aligned}$$

The first-order conditions for an interior solution are

$$\frac{1}{\overline{G}_1(Q_1)}\frac{\partial \Pi(Q_1,Q_2)}{\partial Q_1} = (\overline{p}+\delta-\gamma)G_2(\overline{D}-Q_1)-(c_1-\gamma) \equiv H_1^2(Q_1)=0, \tag{3.12}$$

$$\frac{1}{\overline{G}_2(Q_2)}\frac{\partial \Pi(Q_1,Q_2)}{\partial Q_2} = (\overline{p}+\delta-\gamma)G_1(\overline{D}-Q_2)-(c_2-\gamma) \equiv H_2^2(Q_2)=0. \tag{3.13}$$

By definition, $\{\widehat{Q}_1,\widehat{Q}_2\}$ are the solution of (3.12) and (3.13). Apparently, $\{\widehat{Q}_1,\widehat{Q}_2\} \in (0,\overline{D})$, so they are in Region I or II. By (3.12), $dH_1^2(Q_1)/dQ_1 < 0$. Therefore, for any $Q_1 > \widehat{Q}_1$, $H_1^2(Q_1) < 0$. By symmetry, for any $Q_2 > \widehat{Q}_2$, $H_2^2(Q_2) < 0$. Since in Region I, the optimal order quantity is on the boundary $Q_1+Q_2=\overline{D}$, then if $(\widehat{Q}_1,\widehat{Q}_2)$ is in Region II, then they are the optimal order quantities for the firm. On the other hand, if $(\widehat{Q}_1,\widehat{Q}_2)$ is in Region I, then it can be easily verified that for all the (Q_1,Q_2) in Region II, either $H_1^2(Q_1) < 0$ or $H_2^2(Q_2) < 0$. Therefore, the optimal order quantity is on the boundary $Q_1+Q_2=\overline{D}$. In summary, the optimal order quantities are characterized by (3.11). It can be verified that if there exists an interior optimal solution, then the Hessian at the optimal point is negative definite. Therefore, the profit function is unimodal, and it achieves its maximum at a unique point. ■

Since $\widehat{Q}_1(c_1)+\widehat{Q}_2(c_2) \leqslant \overline{D}$ is equivalent to $G_1^{-1}\left(\frac{c_2-\gamma}{\overline{p}+\delta-\gamma}\right)+G_2^{-1}\left(\frac{c_1-\gamma}{\overline{p}+\delta-\gamma}\right) \geqslant \overline{D}$, the proof of Proposition 3.4.5 follows from Lemmas 3.4.6-3.4.8. ■

Figure 3.1(b) illustrates Proposition 3.4.5 and demonstrates how the changes in the wholesale prices affect the price-taking firm's optimal diversification decision. The dedication zone consists of two areas: I(1) and I(2), where the firm orders only from supplier 1 or supplier 2, respectively. The diversification zone, is subdivided into diversification zones I and II, where the total order quantity is equal to or greater than, respectively, the deterministic demand $\overline{D}$. Moreover, the optimal order quantities in diversification zones I and II are characterized by (3.8) and (3.9), respectively.

When the wholesale prices lie in diversification zone I, the firm's optimal total order quantity is exactly equal to the demand $\overline{D}$. The first equation of (3.8) can be interpreted as the firm's marginal profit $\overline{G}_1(Q_1)(\overline{p}+\delta-c_1)$ from supplier 1, is equal to the firm's marginal profit $\overline{G}_2(Q_2)(\overline{p}+\delta-c_2)$ from supplier 2. In diversification zone II, if supplier i's random capacity r_i turns out to be less than Q_i, then the firm's marginal revenue and cost from supplier i are both zero. On the other hand, conditional on the full delivery of supplier i, the firm's marginal cost is c_i, while its marginal revenue from supplier i depends on the realization of supplier $(3-i)$'s capacity, $i=1,2$. If the random capacity of supplier $(3-i)$ is larger than $\overline{D}-Q_i$, $i=1,2$, then the total delivery will exceed the demand and the marginal revenue is γ; otherwise, the firm cannot meet all the demand and the marginal revenue is $\overline{p}+\delta$. Equating the expected marginal revenue and the expected marginal cost yields equation (3.9).

Proposition 3.4.9 *The effective purchase cost $C_i^T(c_i)$ increases in the market price, the cost of lost goodwill, and the market demand.*

Proof of Proposition 3.4.9. By (3.7), the proof is trivial. ■

When the market demand increases, the optimal order quantity for the single-supplier case increases, and thus the probability for a supplier to deliver the optimal quantity decreases; when the market price or the cost of lost goodwill increases, the firm's marginal revenue increases, which leads to an increase in the opportunity cost of an undelivered unit. Therefore, the imputed cost of a supplier's unreliability increases, which increases the effective purchase cost from him. By Proposition 3.4.5, Proposition 3.4.9 implies that as the market price, the cost of lost goodwill, or the demand increase, the diversification zone becomes larger. However, note that when the market price and the cost of lost goodwill increases, the feasible area for the wholesale prices and the dedication zone become larger as well. Proposition 3.4.10 indicates how the area of the diversification zone as a fraction of the feasible area changes with respect to the market price and the cost of lost goodwill.

Proposition 3.4.10 *The area of the diversification zone as a fraction of the feasible area increases in the market demand, but it does not change in the market price and the cost of lost goodwill.*

Proof of Proposition 3.4.10. Consider the diversification zone above $c_2 = c_1$. For any fixed value of c_1, it is sufficient to show that (the distance between the curves $c_2 = C_1^T(c_1)$ and $c_2 = c_1$)/(the distance between the curves $c_2 = \overline{p} + \delta$ and $c_2 = c_1$), i.e. $(\overline{p} + \delta - c_1)G_1(\overline{D})/(\overline{p} + \delta - c_1) = G_1(\overline{D})$ increases in $\overline{D}$ and does not depend on $\overline{p}$ and δ, which is straightforward. The proof is similar for the diversification zone below $c_2 = c_1$. ■

From Proposition 3.4.10, unlike for the price-setting firm, the economic parameters have no impact on the price-taking firm's propensity to supply diversification. It is determined only by the market demand and suppliers' reliabilities. Next, we examine how the economic parameters affect the price-taking firm's order quantities.

Proposition 3.4.11 (i) *The optimal order quantity for each supplier increases in the market demand.* (ii) *In diversification zone* I*, the optimal order quantity for the more-expensive (resp., less-expensive) supplier increases (resp., decreases) in the market price and the cost of lost goodwill.* (iii) *In diversification zone* II*, the optimal order quantity for each supplier increases in the market price, the cost of lost goodwill, and the salvage value.*

Proof of Proposition 3.4.11. (i) The proof for the dedication zone is trivial. In the diversification zone I,

$$\frac{d\overline{Q}_1^T}{d\overline{D}} = \frac{(\overline{p} + \delta - c_2)g_2(\overline{D} - \overline{Q}_1^T)}{(\overline{p} + \delta - c_1)g_1(\overline{Q}_1^T) + (\overline{p} + \delta - c_2)g_2(\overline{D} - \overline{Q}_1^T)} > 0,$$
$$\frac{d\overline{Q}_2^T}{d\overline{D}} = \frac{(\overline{p} + \delta - c_1)g_1(\overline{Q}_1^T)}{(\overline{p} + \delta - c_1)g_1(\overline{Q}_1^T) + (\overline{p} + \delta - c_2)g_2(\overline{D} - \overline{Q}_1^T)} > 0.$$

In the diversification zone II, for $i = 1, 2$, $d\widehat{Q}_i^T/d\overline{D} = 1 > 0$.

(ii) Without loss of generality, assume $c_1 > c_2$. In the diversification zone I, by (3.8), $\overline{G}_1(\overline{Q}_1^T) - \overline{G}_2(\overline{D} - \overline{Q}_1^T) > 0$. Thus,

$$\frac{d\overline{Q}_1^T}{d\overline{p}} = -\frac{d\overline{Q}_2^T}{d\overline{p}} = \frac{d\overline{Q}_1^T}{d\delta} = -\frac{d\overline{Q}_2^T}{d\delta} = \frac{\overline{G}_1(\overline{Q}_1^T) - \overline{G}_2(\overline{D} - \overline{Q}_1^T)}{(\overline{p} + \delta - c_1)g_1(\overline{Q}_1^T) + (\overline{p} + \delta - c_2)g_2(\overline{D} - \overline{Q}_1^T)} > 0.$$

(iii) In the diversification zone II, by (3.9), it can be verified that, for $i = 1, 2$, $d\widehat{Q}_i^T/d\overline{p} > 0$, $d\widehat{Q}_i^T/d\delta > 0$, and $d\widehat{Q}_i^T/d\gamma > 0$. ■

Intuitively, as the market demand increases, the firm orders more from each supplier. In diversification zone II, as the market price, the cost of lost goodwill, or the salvage value increases, the firm's expected marginal revenue increases. Therefore, the firm orders more from each supplier. Interestingly, in diversification zone I, i.e., when both supplier's wholesale prices are relatively high, as the market price or the cost of lost goodwill increases, the firm will shift some orders from the less-expensive supplier to the more-expensive supplier without changing the total order quantity. The intuition behind this result is as follows. Since the optimal total order quantity is fixed in diversification zone I, the firm must balance the expected marginal profits from the two suppliers. If $c_1 > c_2$, then the firm's marginal profit $\overline{p} + \delta - c_1$ from supplier 1, is less than the firm's marginal profit $\overline{p} + \delta - c_2$ from supplier 2. To balance the expected marginal profits, the probability $\overline{G}_1(Q_1)$ of getting supplier 1's marginal profit must be larger than the probability $\overline{G}_2(Q_2)$ of getting supplier 2's marginal profit. Therefore, an increase of $\overline{p}$ or δ will make the firm's expected marginal profit from supplier 1 higher than that from supplier 2. To counterbalance this effect, the firm has to reduce the probability of getting supplier 1's marginal profit by increasing the order quantity from him. The implications for the suppliers are as follows. When both supplier's wholesale prices are relatively low, an increase in the market price or the cost of lost goodwill is beneficial to both suppliers. However, when both supplier's wholesale prices are relatively high, an increase of the market price or the cost of lost goodwill is only beneficial to the more expensive supplier, while the cheaper supplier will gain with a decrease of the market price or the cost of lost goodwill.

3.4.3 Comparison of Diversification Zones

In this subsection, we compare the diversification zones of the price-setting and price-taking firms. By (3.4) and (3.7), it is easy to verify that the effective purchase costs defined for both types of firms are increasing in the supplier's wholesale price. Note that the increase rates of both effective purchase costs are less than 1, which indicates that the imputed cost of supplier's unreliability is decreasing in his wholesale price. Moreover, as can be seen from

Figure 3.1, the effective purchase cost is convex in the supplier's wholesale price for the price-setting firm, while it is linear for the price-taking firm.

As in Figure 3.1, a firm's diversification zone depends on the effective purchase costs from its suppliers. To compare the diversification zones in the two pricing power models, we make the following assumptions:

$$\overline{p} + \delta = \frac{a}{b}, \qquad G_i(\overline{D}) \geqslant \frac{1}{A}\int_0^A G_i(r_i)dr_i, \qquad \text{for}\, i = 1, 2.$$

The first assumption ensures that the feasible area for the suppliers' wholesale prices is the same for both models, which can be clearly seen in Figure 3.1. The second assumption requires that the market demand is reasonably large. With these assumptions, we present the following proposition which compares the price-setting firm's diversification zone with the price-taking firm's.

Proposition 3.4.12 *The price-setting firm's diversification zone is smaller than the price-taking firm's.*

Proof of Proposition 3.4.12. When $\overline{p} + \delta = a/b$, with the assumption that $G_i(\overline{D}) \geqslant \frac{1}{A}\int_0^A G_i(r_i)dr_i$, it can be verified that $C_i^T(\overline{p}+\delta) = C_i^S(a/b)$ and $C_i^T(\gamma) \geqslant C_i^S(\gamma)$ for $i = 1, 2$. Furthermore, $C_i^S(c_i)$ is increasing and convex in c_i, while $C_i^T(c_i)$ is increasing and linear in c_i. Therefore, it is straightforward to show that $C_i^T(c_i) > C_i^S(c_i)$ for $c_i \in (\gamma, a/b)$, which implies that the price-setting firm's diversification zone is smaller than the price-taking firm's. ■

Intuitively, both pricing and supply diversification help a firm to hedge supply uncertainty. Therefore, the price-setting firm's need for diversification is naturally smaller.

3.4.4 Impact of Supplier Reliability on Effective Purchase Cost

In order to investigate the impact of supplier reliability, we first define a supplier's reliability using some concepts of stochastic dominance that can be found in Shaked and Shanthikumar (2007). A random variable Y is said to be greater than another random variable X in the usual stochastic order, denoted as $Y \geqslant_{st} X$, if $E[f(Y)] \geqslant E[f(X)]$ for all increasing functions f. Likewise, Y is said to be greater than X in the convex order, denoted as $Y \geqslant_{cx} X$, if $E[f(Y)] \geqslant E[f(X)]$ for all convex functions f. Dada et al. (2007) define a supplier as more reliable if his capacity increases in the usual stochastic order; in this paper, we will say that he becomes *first-order more reliable*. It is important to note that this definition ignores the capacity "variability" when comparing a supplier's reliability with different capacities. In practice, however, it is common to view a supplier to also become more reliable if his capacity variability decreases with the capacity mean remaining unchanged. This motivates us to define another notion of reliability based on the convex order stochastic dominance. Accordingly, a supplier with capacity Y is *second-order more reliable* than that with capacity X if $Y \leqslant_{cx} X$. Together, these two notions of reliability cover increases in a supplier's reliability brought about by having a "larger" capacity or a "less variable" capacity. For normally distributed capacities, the relatively abstract notions of reliability can be made concrete: a supplier with capacity Y is first-order more reliable than that with capacity X if $E[Y] \geqslant E[X]$ and $Var(Y) = Var(X)$ and he is second-order more reliable if $E[Y] = E[X]$ and $Var(Y) \leqslant Var(X)$.

In view of the importance of the effective purchase cost from a supplier on a firm's diversification decision, we study how the changes in a supplier's reliability affect the effective purchase cost from that supplier. First, we examine the case for the price-setting firm.

Proposition 3.4.13 *The price-setting firm's effective purchase cost $C_i^S(c_i)$ from supplier i, decreases as he becomes first-order or second-order more reliable.*

Proof of Proposition 3.4.13. By the definition of the usual stochastic order, when the capacity of supplier i increases in the usual stochastic order, its distribution $G_i(r)$ decreases (Shaked and Shanthikumar 2007, 1.A.1, p.3). It follows from (3.4) that the effective purchase cost from a supplier decreases as its capacity increases in the usual stochastic order. With some algebra, we can rewrite the effective purchase cost from supplier i as

$$C_i(c_i) = c_i + \frac{2}{b} \int_0^{\frac{a-bc_i}{2}} G_i(r)dr.$$

If the capacity of supplier i decreases in the convex order, then it follows from the property of convex order that $\int_0^a G_i(r)dr$ will decrease for any a ((Shaked and Shanthikumar 2007, 3.A.8, p.110). Thus the effective purchase cost from a supplier decreases as its capacity decreases in the convex order. ■

Based on Proposition 3.4.13, with the price-setting firm, if a supplier's capacity is normally distributed, he can reduce the effective purchase cost by either increasing the capacity mean or reducing the capacity variance. This result has significant implications for the suppliers. Recall that, in order to become the sole supplier to the firm, the low cost supplier should reduce the effective purchase cost from him (Corollary 3.4.1). Proposition 3.4.13 offers a prescription on how to reduce the effective purchase cost from him: A supplier can achieve this not only by purchasing more equipment or hiring more employees, but also by reducing capacity variability using, for example, the popular Six Sigma methodology.

Next, we examine the case for a price-taking firm. Unlike in the case of the price-setting firm, the price-taking firm's effective purchase cost from a supplier is not monotone with respect to his second-order reliability. For our analysis, we assume the suppliers' capacities to satisfy the property that the CDFs of any two convex-ordered random capacities cross exactly once. The single-crossing property holds in most practical applications. For example, it holds when the two convex-ordered capacities belong to the same member of the location-scale distribution class or the same member of the generalized location-scale distribution class. A distribution $F(x, \mu, \sigma)$ with parameter (μ, σ) is a location-scale distribution if $F(x, \mu, \sigma) = \Phi((x-\mu)/\sigma)$. Examples of location-scale distributions include normal, gamma, Cauchy, Weibull, t, stable, F, Laplace, extreme value, logistic, beta, uniform, and triangular. $F(x, \mu, \sigma)$ is a generalized location-scale distribution if $F(x, \mu, \sigma) = \Phi((\kappa(x) - \mu)/\sigma)$, where $\kappa(\cdot)$ is an increasing function such as the logarithmic function. The generalized location-scale distributions include lognormal. See, for example, Zhang (2005) for further discussions on these two families of distributions.

Proposition 3.4.14 *For the price-taking firm,* (i) $C_i^T(c_i)$ *decreases as* R_i *increases in the* *usual stochastic order.* (ii) *Consider two possible random capacities R_i^1 and R_i^2 for supplier i with the corresponding effective purchase costs $C_i^{T1}(c_i)$ and $C_i^{T2}(c_i)$, respectively. Assume that $R_i^1 \leqslant_{cx} R_i^2$ and their distributions cross at μ_i. Then, if $\overline{D} \leqslant \mu_i$, we have $C_i^{T1}(c_i) \leqslant C_i^{T2}(c_i)$; otherwise, $C_i^{T1}(c_i) > C_i^{T2}(c_i)$.*

Proof of Proposition 3.4.14. The first claim follows from the definition of usual stochastic order and (3.7). Define $G_i^1(\cdot)$ and $G_i^2(\cdot)$ as the CDFs of R_i^1 and R_i^2 respectively. Then, $G_i^1(D) \geqslant G_i^2(D)$ for $D \geqslant \mu_i$ and $G_i^1(D) \leqslant G_i^2(D)$ for $D \leqslant \mu_i$ (see, for example, Wolfstetter 1999, Proposition 4.6, P.143). The second claim follows. ■

In words, the price-taking firm's effective purchase cost from a supplier decreases as his first-order reliability increases. Furthermore, the effective purchase cost decreases (resp., increases) as his second-order reliability increases provided that the demand $\overline{D} \leqslant \mu_i$ (resp., $\overline{D} \geqslant \mu_i$).

It is interesting to compare and contrast Proposition 3.4.13 with Proposition 3.4.14. First, in both cases, as the supplier's first-order reliability increases, the effective purchase cost decreases, which implies that the diversification zone becomes smaller. Second, while the price-setting firm's effective purchase cost decreases as the supplier's second-order reliability increases, this relationship holds for the price-taking firm *only* when the market demand is low. Interestingly, when the market demand is high, the price-taking firm's effective purchase cost increases as the supplier's second-order reliability increases. For example, when the random capacity is normally distributed so that the capacity variability is measured by its standard deviation (Shaked and Shanthikumar 2007, 3.A.51, p.137) and convex-ordered capacity distributions cross at the mean, then by Proposition 3.4.14, a supplier's reliability (in the eyes of the firm) increases as his capacity variance decreases only when his capacity mean is greater than the market demand. When the supplier's capacity mean is less than the market demand, the firm's diversification zone becomes smaller when suppliers become more variable, i.e., the firm's propensity to diversify decreases as the supplier's second-order reliability decreases.

3.4.5 Impact of Supplier Reliability on Optimal Order Quantities

In this subsection, we first investigate how the optimal order quantities in the diversification zone are affected by a supplier's wholesale price and reliability, and then examine how the pricing power of a firm affects the impact of the supplier's reliability on the firm's optimal order quantities.

Proposition 3.4.15 *Consider the price-setting firm.* (i) *In diversification zone* I*, the optimal order quantity from a supplier increases as his wholesale price or his rival's reliability decreases, and it decreases as his rival's wholesale price or his reliability decreases.* (ii) *In diversification zone* II*, the optimal order quantity from a supplier increases only when his wholesale price decreases or his rival's reliability decreases.* (iii) *The total order quantity decreases when either supplier increases his wholesale price or reliability.*

Proof of Proposition 3.4.15. For ease of exposition, the superscript S is removed from this proof. We define ρ_i as the indicator of supplier i's reliability in the sense that a higher ρ_i represents a higher level of reliability. When the supplier becomes more reliable as a result of a stochastically larger capacity, that is, $\partial\overline{G}_i(r,\rho_i)/\partial\rho_i > 0$ and $\partial G_i(r,\rho_i)/\partial\rho_i < 0$, we have:

(i) For $i = 1, 2$, from $a - bc_i - 2\overline{Q}_i - 2\int_0^{\frac{a-bc_{3-i}}{2} - \int_0^{\overline{Q}_i}\overline{G}_i(r,\rho_i)dr} \overline{G}_{3-i}(r,\rho_{3-i})dr = 0$. The implicit function theorem gives $\partial\overline{Q}_i/\partial c_i < 0$, $\partial\overline{Q}_i/\partial c_{3-i} > 0$, $\partial\overline{Q}_i/\partial\rho_i > 0$, and $\partial\overline{Q}_i/\partial\rho_{3-i} < 0$.

(ii) From $2\int_0^{A-\widehat{Q}_i} G_{3-i}(r,\rho_{3-i})dr - b(c_i - \gamma) = 0$, we can obtain $\partial\widehat{Q}_i/\partial c_i < 0$ and $\partial\widehat{Q}_i/\partial\rho_{3-i} < 0$.

(iii) By Corollary 3.4.1, $\overline{Q}_1 + \overline{Q}_2 = (a - bc_i)/2 + \int_0^{\overline{Q}_{3-i}} G_{3-i}(r)dr$. By claim (i), when c_{3-i} increases, $\overline{Q}_{3-i}$ decreases. Therefore, $\overline{Q}_1 + \overline{Q}_2$ decreases. When supplier i's reliability increases, $\int_0^{\overline{Q}_{3-i}} G_{3-i}(r)dr$ decreases, and thus $\overline{Q}_1 + \overline{Q}_2$ decreases. By claim (ii), the result for $\widehat{Q}_1 + \widehat{Q}_2$ follows trivially.

When the supplier becomes more reliable by having a smaller capacity variability in the convex order, then, for any a, we have $\partial \int_0^a \overline{G}_i(r, \rho_i)/\partial \rho_i > 0$. The rest can be proved similarly. ■

Claims for diversification zone I are intuitive: the order quantity for a supplier will increase when he reduces his wholesale price or increases his reliability; it will increase when his rival's wholesale price increases or his rival's reliability decreases. However, an interesting phenomenon occurs when (c_1, c_2) belongs to diversification zone II, i.e., when both suppliers' wholesale prices are low. In this case, the order quantity for a supplier is affected by his wholesale price and not his reliability, but his rival's reliability. In other words, in diversification zone II, a supplier cannot receive a larger order by increasing his reliability, but he can be hurt by his rival doing so. Nevertheless, when a supplier increases his reliability, the separating curve $\widehat{Q}_1^S(c_1) + \widehat{Q}_2^S(c_2) = A$ shifts downward accordingly and a point $(\bar{c}_1, \bar{c}_2)$ that was in diversification zone II may turn out to be in diversification zone I, where the supplier can receive a larger order by increasing his reliability. In summary, when both suppliers' wholesale prices are low and close to each other, then a supplier can receive a larger order by increasing his reliability only after he can bring his reliability up to a certain level.

Next, we examine the impacts of supplier's wholesale price and reliability on the price-taking firm's optimal order quantity in the diversification zone.

Proposition 3.4.16 *Consider the price-taking firm.* (i) *In diversification zone* I, *the optimal order quantity from a supplier increases as his wholesale price or his rival's first-order reliability decreases, and it decreases as his rival's wholesale price or his first-order reliability decreases.* (ii) *In diversification zone* II, *the optimal order quantity from a supplier increases when his wholesale price decreases or his rival's first-order reliability decreases.*

Proof of Proposition 3.4.16. (i-ii) By (3.8) and (3.9), it is straightforward to verify the first two claims by using the definition of usual stochastic order. ■

Note that claims (i) and (ii) in Proposition 3.4.15 and Proposition 3.4.16 are similar in terms of the impacts of wholesale price and first-order reliability on the firm's order quantities. In diversification zone II, like in the case for the price-setting firm, the optimal order quantity from a supplier is affected by his wholesale price and not his reliability, but his rival's reliability. Next, we examine how the price-taking firm's order quantities are affected by a supplier's second-order reliability.

Proposition 3.4.17 *For $i = 1, 2$, consider two possible random capacities R_i^1 and R_i^2 for supplier i with the corresponding optimal order quantities Q_i^{T1} and Q_i^{T2} by the price-taking firm, respectively. Assume that $R_i^1 \leqslant_{cx} R_i^2$ and their distributions cross at μ_i.* (i) *If $\mu_i \geqslant \overline{D}$, then in diversification zone* I, *$Q_i^{T1} \geqslant Q_i^{T2}$ and $Q_{3-i}^{T1} \leqslant Q_{3-i}^{T2}$; in diversification zone* II, *$Q_i^{T1} = Q_i^{T2}$ and $Q_{3-i}^{T1} \leqslant Q_{3-i}^{T2}$.* (ii) *If $0 < \mu_i < \overline{D}$, then in diversification zone* I, *$Q_i^{T1} \geqslant Q_i^{T2}$ and $Q_{3-i}^{T1} \leqslant Q_{3-i}^{T2}$ if $\overline{G}_i(\mu_i)(p + \delta - c_i) \leqslant \overline{G}_{3-i}(D - \mu_i)(p + \delta - c_{3-i})$; $Q_i^{T1} < Q_i^{T2}$ and $Q_{3-i}^{T1} > Q_{3-i}^{T2}$, otherwise.* (iii) *If $0 < \mu_i < \overline{D}$, then in diversification zone* II, *$Q_{3-i}^{T1} \leqslant Q_{3-i}^{T2}$ if $(p + \delta - c_{3-i})/(p + \delta - \gamma) \geqslant \overline{G}_i(\mu_i)$; $Q_{3-i}^{T1} > Q_{3-i}^{T2}$, otherwise.*

Proof of Proposition 3.4.17. Define $G_i^1(\cdot)$ and $G_i^2(\cdot)$ as the CDFs of R_i^1 and R_i^2 respectively. Then, $G_i^1(Q) \geqslant G_i^2(Q)$ for $Q \geqslant \mu_i$ and $G_i^1(Q) \leqslant G_i^2(Q)$ for $Q \leqslant \mu_i$ (see, for example, Wolfstetter 1999, Proposition 4.6, P.143). (iii.A) If $\mu_i \geqslant \overline{D}$, since $Q_i < \overline{D}$, we have $G_i^1(Q_i^{T1}) \leqslant G_i^2(Q_i^{T1})$, which implies that $\overline{G}_i^2(Q_i^{T1})/\overline{G}_{3-i}(D - Q_i^{T1}) \leqslant \overline{G}_i^1(Q_i^{T1})/\overline{G}_{3-i}(D - Q_i^{T1}) = (p + \delta - c_{3-i})/(p + \delta - c_i)$. From the fact that $\overline{G}_i(Q_i)/\overline{G}_{3-i}(D - Q_i)$ is decreasing in Q_i, we have $Q_i^{T1} \geqslant Q_i^{T2}$, and $Q_{3-i}^{T1} = \overline{D} - Q_i^{T1} \leqslant \overline{D} - Q_i^{T2} = Q_{3-i}^{T2}$. The results can be proved similarly in the diversification zone II. (iii.B) If $0 < \mu_i < \overline{D}$ and $\overline{G}_i(\mu_i)(p + \delta - c_i) \leqslant \overline{G}_{3-i}(D - \mu_i)(p + \delta - c_{3-i})$, then $Q_i^{T1} \leqslant \mu_i$. Therefore, $G_i^1(Q_i^{T1}) \leqslant G_i^2(Q_i^{T1})$, which implies that $\overline{G}_i^2(Q_i^{T1})/\overline{G}_{3-i}(D - Q_i^{T1}) \leqslant \overline{G}_i^1(Q_i^{T1})/\overline{G}_{3-i}(D - Q_i^{T1}) = (p + \delta - c_{3-i})/(p + \delta - c_i)$. From the fact that $\overline{G}_i(Q_i)/\overline{G}_{3-i}(D - Q_i)$ is decreasing in Q_i, we have $Q_i^{T1} \geqslant Q_i^{T2}$, and $Q_{3-i}^{T1} = \overline{D} - Q_i^{T1} \leqslant \overline{D} - Q_i^{T2} = Q_{3-i}^{T2}$. Other claims can be proved similarly. ∎

By Proposition 3.4.17, how a supplier's second-order reliability affects the price-taking firm's optimal order quantities from him and his rival depends on the relationship between the market demand and a critical value. When the market demand is relatively small, the impacts of this supplier's second-order reliability on the optimal order quantities are the same as his first-order reliability impacts: as this supplier's second-order reliability increases, in diversification zone I, the optimal quantity ordered from this supplier increases and the optimal quantity ordered from his rival decreases; in diversification zone II, the optimal quantity ordered from this supplier does not change and the optimal quantity ordered from his rival decreases. For ease of exposition, we call this phenomenon the *regular effect* and the reverse phenomenon the *irregular effect*. From claims (ii) and (iii) in Proposition 3.4.17, when the market demand is greater than the critical value, both regular and irregular effects exist in diversification zones I and II. In order to exhibit these two effects more clearly, we illustrate Proposition 3.4.17 in Figure 3.2. When $\{\mu_1 \geqslant \overline{D}, \mu_2 \geqslant \overline{D}\}$, only the regular effect exists in the diversification zone, therefore we omit this case in Figure 3.2.

In Figure 3.2, diversification zones I and II in Figure 3.1(b) are further divided into several sub-zones. If a sub-zone in diversification zone I is marked with I_{RR}, then in this sub-zone, the effects of both suppliers' second-order reliabilities on the optimal order quantities are regular. If a sub-zone in diversification zone I is marked with I_{IR}, then in this sub-zone, the effects of supplier 1's second-order reliability on the optimal order quantities are irregular, while the effects of supplier 2's second-order reliability on the optimal order quantities are regular. Sub-zones I_{RI}, I_{II}, II_{RR}, II_{IR}, II_{RI}, and II_{II} are defined similarly.

From Figure 3.2, we can clearly see that the irregular effects of a supplier i's second-order reliability on the optimal order quantities happen only when $\mu_i < \overline{D}$. From Figure 3.2(d), when $\mu_1 + \mu_2 \leqslant \overline{D}$, the effects of both suppliers' second-order reliabilities may be irregular. Note that the boundaries that separate the regular sub-zones from the irregular sub-zones are not affected by the suppliers' second-order reliabilities. When $\mu_i < \overline{D}$, in diversification zone I, whether the effects of supplier i's second-order reliability are regular depends on the relative magnitude of both suppliers' wholesale prices. Specifically, the effects of supplier i's second-order reliability are regular if supplier i's wholesale price is relatively higher than his rival's; otherwise, the effects are irregular. In diversification zone II, whether the effects of supplier i's second-order reliability are regular depends only on his rival's wholesale price. Specifically, the effects of supplier i's second-order reliability are regular if his rival's wholesale price is low; otherwise, the effects are irregular.

It is interesting to compare and contrast Proposition 3.4.17 with Proposition 3.4.15. Note that the firm's pricing power does not change the effects of a supplier's first-order reliability on the optimal order quantities. However, the way a supplier's second-order reliability affects the optimal order quantities depends on the firm's pricing power. As a result, a supplier has to consider the firm's pricing power when he considers strategies in improving his reliability to win a larger order from the firm. When the firm is able to adjust the retail price in response to supply uncertainty, a supplier should always try to reduce his capacity variability to win a larger order from the firm. However, such an intuitive approach is not necessarily the best for the suppliers when the retail price is given. Figure 3.2 provides a guidance for the suppliers. In Figure 3.2(d), for example, when the suppliers' wholesale prices fall into sub-zone I_{IR}, to win a larger order from the firm, supplier 1 should increase his capacity variability while supplier 2 should decrease his capacity variability. By Propositions 3.4.14 and 3.4.17, a low second-order reliability may benefit the supplier in terms of winning orders from the firm. In other words, the supplier's second-order reliability may be different in the eyes of the buyers with different pricing powers.

So far, we have examined the effects of the suppliers' reliabilities on the effective purchase costs that bound the firm's diversification zone as in Figure 3.1 and on the optimal order quantities within each diversification zone. Next, we examine within the firm's diversification zone, how do the suppliers' reliabilities affect the curve that separates diversification zones I and II. By Corollary 3.4.1 and Proposition 3.4.5, the curve is $\widehat{Q}_1^S(c_1) + \widehat{Q}_2^S(c_2) = A$ for the price-setting firm and it is $G_1^{-1}\left(\frac{c_2-\gamma}{\overline{p}+\delta-\gamma}\right) + G_2^{-1}\left(\frac{c_1-\gamma}{\overline{p}+\delta-\gamma}\right) = \overline{D}$ for the price-taking firm. To study the impact, without loss of generality, we consider c_1 as the independent variable and c_2 as the dependent variable for these two curves.

Proposition 3.4.18 *With the price-setting firm, for* $i = 1, 2$, $\{c_2(c_1) : \widehat{Q}_1^S(c_1) + \widehat{Q}_2^S(c_2) =$

$A\}$ decreases as supplier i becomes more reliable in the first or the second order.

Proof of Proposition 3.4.18. We first examine how the curve is affected by supplier 1's reliability. By Proposition 3.4.15, when c_1 and supplier 2's reliability are fixed, $\widehat{Q}_1^S(c_1)$ is fixed, which means $\widehat{Q}_2^S(c_2)$ has to remain unchanged to ensure that the equality holds. However, by Proposition 3.4.15, $\widehat{Q}_2^S(c_2)$ decreases as c_2 or supplier 1's reliability increases. So, when supplier 1's reliability increases, c_2 must decrease on this curve. Similarly, when supplier 2's reliability increases, c_2 decreases on this curve. ∎

Proposition 3.4.18 implies that, as a supplier's first-order or second-order reliability increases, the price-setting firm's diversification zone II becomes smaller and the firm's propensity to order more than the abundant supply decreases.

Proposition 3.4.19 *With the price-taking firm,* (i) *the curve $\left\{c_2(c_1) : G_1^{-1}\left(\frac{c_2-\gamma}{\overline{p}+\delta-\gamma}\right) + G_2^{-1}\left(\frac{c_1-\gamma}{\overline{p}+\delta-\gamma}\right) = \overline{D}\right\}$ decreases as supplier i's first-order reliability increases for $i = 1, 2$;* (ii) *as supplier 1's second-order reliability increases, the portions of this curve that separate sub-zones I_{RR}&II_{RR} and I_{RI}&II_{RI} decrease while the portions of this curve that separate sub-zones I_{IR}&II_{IR} and I_{II}&II_{II} increase. The impacts of supplier 2's second-order reliability are symmetric to supplier 1's.*

Proof of Proposition 3.4.19. (i) Consider the supplier's first-order reliability only. The proof is similar to the proof of Proposition 3.4.18 by using the results of Proposition 3.4.16. (ii) We first examine how the curve is affected by supplier 1's second-order reliability. By claim (ii) of Proposition 3.4.17, when c_1 and supplier 2's reliability are fixed, $\widehat{Q}_1^T(c_1)$ is fixed, which means $\widehat{Q}_2^T(c_2)$ has to remain unchanged to ensure that the equality holds. However, by Proposition 3.4.17, in sub-zone II_{RR} or II_{RI}, $\widehat{Q}_2^T(c_2)$ decreases as c_2 or supplier 1's second-order reliability increases. So, when supplier 1's second-order reliability increases, c_2 must decrease on this curve. On the other hand, in sub-zone II_{IR} or II_{II}, $\widehat{Q}_2^T(c_2)$ increases as c_2 decreases or supplier 1's second-order reliability increases. So, when supplier 1's second-order reliability increases, c_2 must increase on this curve. The impacts of supplier 2's second-order reliability can be proved similarly. ∎

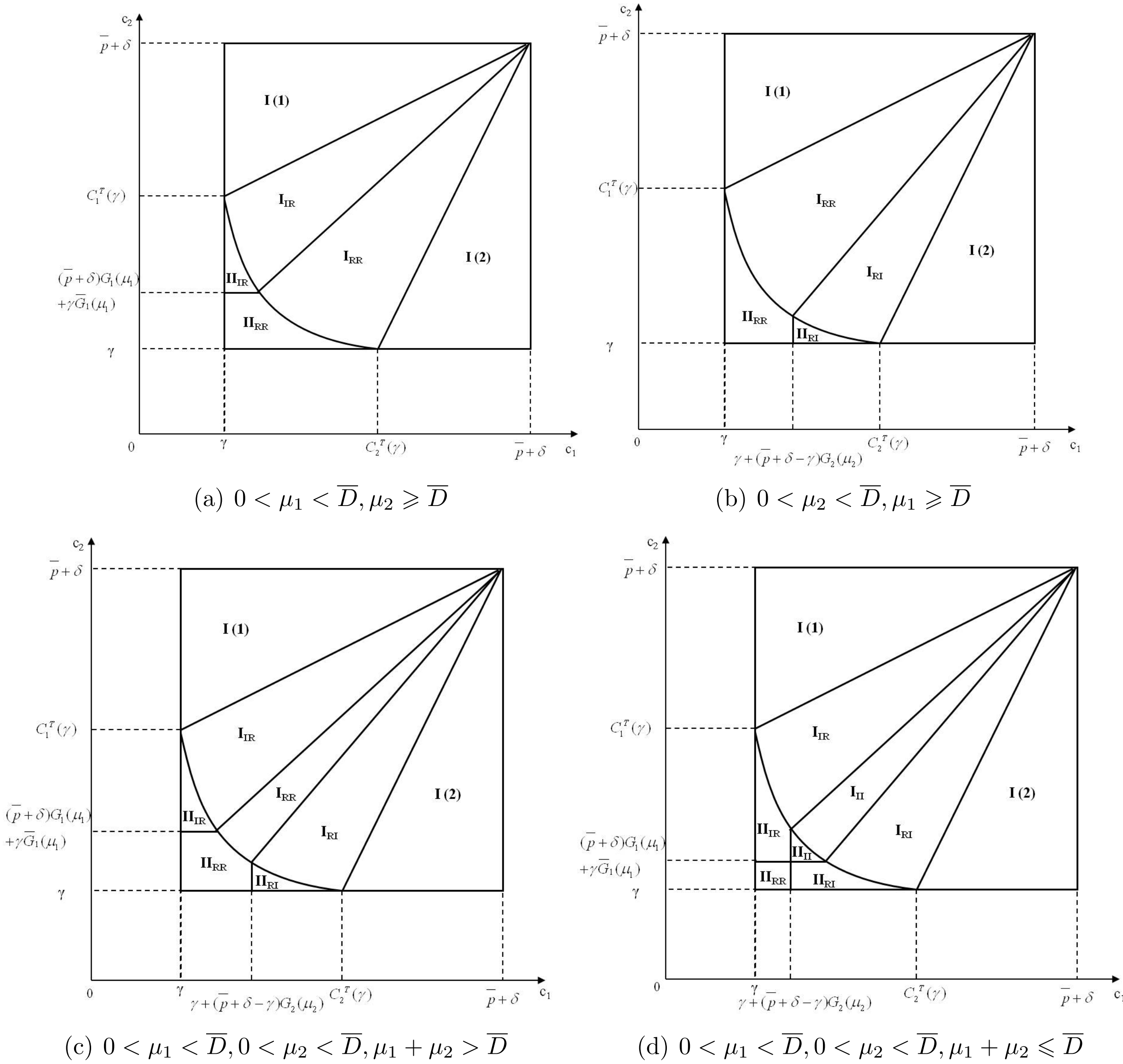
Comparing and contrasting Proposition 3.4.19 with Proposition 3.4.18, we see that the effects of the suppliers' first-order reliabilities on deciding whether the firm should order more than the abundant supply do not depend on the firm's pricing power while the effects of the second-order reliability do. For the price-taking firm, the effects of the suppliers' second-order reliabilities on deciding whether the firm should order more than the abundant supply depends on whether the effects of the suppliers' second-order reliabilities on the optimal order quantities are regular or irregular. Specifically, if the effects of supplier i's second-order reliability on the optimal order quantities are regular (irregular, resp.), then the firm's propensity to order more than the abundant supply decreases (increases, resp.) as supplier i's second-order reliability increases.

3.5 Concluding Remarks

We consider the diversification problems of price-setting and price-taking firms with two unreliable suppliers having random capacities. The related literature considering an exogenous price and the first-order reliability derives the insight that cost is the order qualifier while reliability is the order winner, when picking suppliers and deciding on order quantities. The insight continues to hold in terms of both first- and second-order reliabilities for the price-setting firm. However, the insight does not hold in terms of the second-order reliability for the price-taking firm. These results have important implications for a supplier who wants to win a larger order by adjusting his capacity. With a price-setting firm, a supplier benefits from his efforts in reducing his capacity variability. On the other hand, with a price-taking firm, a supplier may, in some cases, lose orders from the firm when his capacity variability is reduced.

Interestingly, we find that, regardless of a firm's pricing power, when the wholesale prices of the suppliers are low, the optimal ordered quantity from a supplier is *not* affected by his reliability or his rival's price, but only affected by his own price and his rival's reliability. In this case, the suppliers can receive larger orders by increasing (or decreasing) their reliabilities only beyond a threshold reliability level.

We have made several assumptions in this paper to keep the analysis tractable. We have only considered one specific demand form in this paper. We expect that our results would carry over to more general demands even though the expression of the effective purchase cost would be more involved. To focus on the effect of supply uncertainty on the firm's sourcing decisions, we have assumed a deterministic demand. It would be of interest to see if the main insights developed in this paper continues to hold for stochastic demands.



(a) $0 < \mu_1 < \overline{D}, \mu_2 \geqslant \overline{D}$

(b) $0 < \mu_2 < \overline{D}, \mu_1 \geqslant \overline{D}$

(c) $0 < \mu_1 < \overline{D}, 0 < \mu_2 < \overline{D}, \mu_1 + \mu_2 > \overline{D}$

(d) $0 < \mu_1 < \overline{D}, 0 < \mu_2 < \overline{D}, \mu_1 + \mu_2 \leqslant \overline{D}$

Figure 3.2. Effects of the Second-Order Reliability on a Price-Taking Firm's Optimal Order Quantities

CHAPTER 4

STRATEGIC INVENTORIES AND DYNAMIC COORDINATION WITH PRODUCTION COST LEARNING

4.1 Synopsis

In this chapter, we consider a vertical supply chain framework with one manufacturer and one retailer. We focus on the impacts of the stochastic learning curve and strategic inventory on these two players' strategic and operational decisions. In particular, we are interested in answering the following questions: (a) How to coordinate the two-period supply chain with stochastic learning? (b) How does the stochastic learning curve affect the value of strategic inventory to the retailer, the manufacturer, and the supply chain? (c) How are these results affected by the holding cost, market potentials, and learning efficiency?

We consider a two-period learning curve model in which the second-period production cost decreases in the first-period production quantity with some random variation in the learning rate. We assume that the market potentials in the two periods are arbitrary and not necessarily of the same size. This generalization allows us to study the impact of relative magnitude of market potentials on the value of strategic inventory. The manufacturer and the retailer know the probability distribution of the stochastic learning factor at the beginning of the first period and the uncertainty of the learning factor is completely resolved at the end of the first period. To explicitly study the value of strategic inventory, we first consider the case in which inventory is not allowed to be carried over to the second period. This situation arises, for example, when the retailer does not have a warehouse to store the inventory. We then study the case in which inventory is allowed to be carried over to satisfy the second-period demand. In each case, we characterize the sub-game perfect equilibrium pricing and ordering decisions. In the end, we compare the channel members' profits with and without inventory carryover option and identify the preferred cases for the retailer, the manufacturer, and the supply chain.

A decentralized supply chain suffers from double marginalization effect and produces less than the optimal quantity. Suboptimal production results in insufficient cost reduction as the learning curve is not fully utilized. The higher is the potential of cost learning, the more severe is the double marginalization. Therefore, we expect that double marginalization becomes more severe in the context of learning. As expected, we find that the retail price in each period is higher in the decentralized channel than that in the centralized channel; production quantities and supply chain profits are lower in the decentralized channel than those in the centralized channel. Furthermore, the changes in retail prices, production quantities, and profits increase in the average learning rate and the uncertainty in the learning rate. Revenue-sharing (RS) contract is known to coordinate the static supply chain and therefore achieve full channel efficiency. We investigate whether RS contracts coordinate the two-period dynamic supply chain and if so, how the contract parameters and production learning efficiency parameter affect the coordinating contracts and the split of profits between supply chain members in the two periods.

We first present the feedback equilibrium prices and quantities and coordinating RS contracts assuming no inventory carry-over option. The coordinating RS contracts require that the wholesale price in Period 2 to be proportional to the production cost in Period 2 while the coordinating wholesale price in Period 1 to be a function of the two revenue-sharing rates of both periods. The two-period RS contracts bring more flexibility to the channel members when negotiating over the contract parameters. We then generalize the model by allowing the units left over from the first period to meet demands in the second period. Even when allowed to carry inventories, the retailer will do so only if certain conditions are satisfied. We compare the results with inventory carryover option with those without the option. We also investigate the coordinating RS contracts under inventory-carryover and compare the contracts with and without inventory carry-over option. We find that when inventory is allowed to be carried over, the manufacturer has less flexibility in selecting the coordinating RS contracts.

We show that when inventories are allowed, in equilibrium, they are carried under certain conditions. When the market is symmetric, inventories are not carried in a centralized channel while they may be carried in a decentralized channel. Consider a special case when the learning curve effect is absent and market is symmetric. Inventories are not carried even though they are allowed while they are carried in the decentralized channel when the inventory holding cost is low. The manufacturer's profit is always higher when inventories are allowed than that when they are not. However, depending on the holding costs, the retailer's and supply chain's profits may be greater or smaller with inventories. For a low inventory holding cost, the retailer's and channel's profits are greater when inventories are allowed than those when inventories are not allowed. For a medium inventory holding cost, the retailer and supply chain are better off when inventories are not allowed to be carried over than when they are. Two questions arise at this point. (a) Why would the retailer carry inventories when he would be better off when inventories are not allowed? (b) Why are the retailer and supply chain worse off with inventories while the manufacturer is always better off with inventories? This chapter addresses these questions.

The rest of the paper is organized as follows. Section 4.2 provides a survey of related research. Section 4.3 introduces the assumptions and setup of the model. Section 4.4 provides the model in which the inventory is not allowed to carry over from Period 1 to Period 2. We allow the inventory carry-over in section 4.5. Section 4.6 examines the value of strategic inventory. Section 4.7 provides concluding remarks.

4.2 Literature Review

The phenomenon of *learning-by-doing*, i.e., the reduction of production cost through repeated production, has been well observed in many industries. An early work by Wright (1936) found that the direct labor cost fell by 20% with every doubling cumulative production in the aerospace industry. Subsequent studies have shown that learning curve phenomena exists in various industries (Baloff 1971, Yelle 1979, Hatch and Mowery 1998). A few papers have focused on the economic impacts of spillover learning effects on market structure, competitive behavior, capital investment, trade policies, and intertemporal externality in production (Arrow 1962, Spence 1981, Fudenberg and Tirole 1983, Dasgupta and Stiglitz 1988). Applications of the learning curve in the operations management include optimal production planning under uncertainty (Mazzola and McCardle 1996, Mazzola and McCardle 1997, Majd and Pindck 1989, Özer and Uncu 2012), inventory and production decisions under joint demand and cost learning (Jøgensen et al. 1999), capacity expansion (Hiller and Shapiro 1986), lot sizing (Karwan et al. 1988, Chand and Sethi 1990, Tzur 1993).

To capture the dynamic nature of production cost learning, we consider a two-period model with linear demand and learning in which the second-period production cost decreases linearly in the first-period production quantity with uncertainty in the efficiency of learning, i.e., a random learning rate. A few papers have incorporated uncertainty into the learning process. Mazzola and McCardle (1996, 1997) introduce random variation in the (exponentially) decreasing cost models. Mazzola and McCardle (1996) show that results with stochastic learning differ from those with deterministic learning: in some stochastic models, the optimal production exceeds the myopic production, a key result from the deterministic learning literature; in other stochastic models, this result does not hold. Mazzola and McCardle (1997) combine two types of learning: cost learning through a learning curve with a random learning rate and Bayesian learning regarding the parameters of the cost function. They show that the result – optimal production increases with cumulative production – does not hold in the presence of Bayesian learning. Alvarez and Amman (1999) assume a stochastic learning-by-doing process and focus on estimating the stochastic cost structure during the production process. There are linear learning models in which the production costs (or the deterministic component of cost) are linearly decreasing in the production volume (Fudenberg and Tirole 1983, Alvarez and Amman 1999). Others have used nonlinear models in which the (deterministic component of) costs decrease exponentially with cumulative production (Arrow 1962, Baloff 1971, Mazzola and McCardle 1996, 1997).

Most of the previous research in the stream of learning curve studies the implications and policies from a monopolistic manufacturing firm's perspective. We extend the applications of learning curve effect to a dynamic and decentralized vertical supply chain. We are particularly interested in the impacts of learning curve on *strategic* inventory and dynamic channel coordination. Strategic inventories are carried not for operational reasons but for strategic considerations. Previous research has shown that strategic inventories play important role in horizontal and vertical competitions. Saloner (1996) considers a two-period duopoly model in which firms make their production decisions in the first period and their sales decisions in the second period. Firms may carry unsold units from the first period to satisfy the second-period demand. The first-mover advantage cannot be achieved if the firm cannot credibly commit to selling its entire production in the first period. Inventories serve as a commitment to achieve the first-mover advantage. Rotemberg and Saloner (1986) consider a duopoly model in which strategic inventories are used to sustain collusive profits by the threat of reversion to competitive behavior. These two papers study strategic inventories in horizontal competition. Anand et al. (2008) study vertical competition in a decentralized two-period supply chain with one buyer and one supplier. By carrying strategic inventories, the buyer induces supply-side competition between the supplier and buyer's inventories in the second period. Therefore, the buyer effectively forces the supplier to lower the second-period wholesale price, which reduces the level of double marginalization. Erhun et al. (2008) study the dynamic pricing/procurement strategy with both market demand uncertainty and supply capacity constraint in a two-period model setup.

We show that with the presence of learning curve effect, the double marginalization problem gets more severe, which motivates the dynamic supply chain coordination. Although there are a vast number of papers studying the supply chain coordination with static models, very few papers investigate the supply chain coordination in dynamic environments (Lee et al. 2000, Taylor 2001, Linh and Hong 2009, Oh and Özer 2012). The dynamics in Lee et al. (2000) and Taylor (2001) come from declining retail prices. Assuming two buying opportunities and exogenous wholesale prices, Lee et. al. (2000) show that price protection cannot guarantee channel coordination. Taylor (2001) examines price protection and returns policies as channel coordination mechanisms. He finds that end-of-life returns achieve channel coordination but the manufacturer is worse off. The combination of policies of price protection and returns can result in both the channel coordination and a win-win situation for channel members. Linh and Hong (2009) consider a two-period newsvendor supply chain and show that revenue sharing contracts can coordinate the dynamic supply chain. Revenue sharing contracts have been widely used in the operations management literature and are known to coordinate the static supply chains in one-period setup (Gerchak and Wang 2004, Cachon and Lariviere 2005, Gerchak et al. 2006). In this paper, we use revenue sharing contracts to coordinate a decentralized supply chain with stochastic production cost learning curve. We find that revenue sharing contracts coordinate the two-period supply chain whether inventories are allowed or not. While Lihn and Hong assume the same revenue sharing rate for two periods, we allow the sharing rates to be general. When inventories are carried, the revenue sharing contracts under such situations are less flexible than when inventories are not allowed or not carried. We also provide insights on the trade-off between the revenue sharing rate and the coordinating wholesale price in each period.

4.3 A Two-Period Model Setup

Consider a supply chain in which the manufacturer produces a product and sells it through a retailer in multiple selling seasons (periods). Due to the learning curve effect, the manufacturer's per unit production cost declines with previous production experience. This assumption is particularly appropriate for the industries that are in the infant stage. When the industry reaches the mature stage, the learning effect may become less significant and even disappear.

To explicitly study the dynamic nature of production cost learning, we assume there are two production and selling periods. In practice, production can occur in many periods. Our learning curve model can be generalized for any finite number of periods. Two-period models have been used widely in the learning curve literature (for example, Spence 1981, Fudenberg and Tirole 1983, and Dasgupta and Stiglitz 1988). The manufacturer incurs a production cost of c_i in Period i, $i = 1, 2$. The second period production cost decreases linearly in the production quantity in the first period with some uncertainty in the learning rate, i.e., the manufacturer is not certain about at which rate he can reduce the production cost in Period 2. Mazzola and McCardle (1997) consider a stochastic model of learning curve with random variation whose distribution is known. Mazzola and McCardle (1996) also study the stochastic learning curve effect but the firm has uncertainty regarding the distribution of the variation in production costs. While the random variation is additive in their log-linear model, the variation in our model comes from the learning rate. Let $\tilde{\gamma} = \gamma x$ denote the stochastic learning rate, where γ is the deterministic component of the learning rate that captures the efficiency of learning and $x \in [0, 1]$, is the random component. The random variable x has a probability density function (pdf) of $f(x)$ with a mean $\mu \equiv \int_0^1 x f(x) dx$ and a variance $\sigma^2 \equiv \int_0^1 x^2 f(x) dx - \mu^2$. Therefore, the second period production cost can be written as $c_2(x) = c_1 - \tilde{\gamma} q_1 = c_1 - x\gamma q_1$, where q_1 is the first-period production quantity. Accordingly, we have the expected learning rate $E[\tilde{\gamma}] = \mu\gamma$ and the expected second-period production cost $E[c_2(x)] = c_1 - \mu\gamma q_1$. The higher the μ or γ is, the faster the manufacturer can learn (i.e., the more efficient in production cost reduction). The uncertainty in the learning rate is completely resolved at the end of first period.

The retailer's problem is to decide the order quantity and the retail price in each period. The manufacturer's problem is to decide the wholesale price in each period. Wholesale and retail prices and the order quantities are determined in a Stackelberg game in which the manufacturer is the leader and the retailer the follower. Motivated by the declining retail prices and manufacturing cost in the technology-related industries, Lee et. al. (2000) and Taylor (2001) allow the retailer to have two ordering opportunities but retail prices are exogenously set.

The supply chain operates in a make-to-order environment: at the beginning of period

i ($i = 1, 2$), the retailer orders q_i units of the product and the manufacturer produces to satisfy the retailer's orders. Let p_i be the retailer's selling price in Period $i = 1, 2$. The sales in Period i are $D_i(p_i) = a_i - bp_i$, where a_i is the market potential in Period $i = 1, 2$, and b captures the customers' price sensitivity to the retail price p_i. Anand et al.(2008) use a similar linear demand function but assume the market potentials are symmetric, i.e., $a_1 = a_2$.

We first assume that inventories are not allowed to be carried over so the retailer orders exactly to satisfy the current period's demand, i.e., $q_i = D_i(p_i), i = 1, 2$. We then relax this assumption and allow the unsold units from the first period to satisfy the demand in the second period. In the latter case, the retailer may order more than the first period demand, i.e., $q_1 > D_1(p_1)$ and carry the leftover units $I_2 = q_1 - D_1(p_1)$ to the second period. We assume that the unmet demands are lost in each period.

We assume that the manufacturer and the retailer are both *forward-looking* (far-sighted) profit maximizers. In the first period, the manufacturer and the retailer each maximizes the total profit obtained from both periods. In a two-period model, one important assumption is that when the wholesale prices w_1 and w_2 are both announced in the first period, whether the manufacturer commits to the announced prices. Anand et al. (2008) study both the dynamic contract and commitment contract. Under the latter contract, the supplier quotes wholesale prices w_1 and w_2 at the beginning of the horizon. In this chapter, we focus on the dynamic contract under which the manufacturer sequentially announces wholesales prices w_1 and w_2 at the beginning of each period. Correspondingly, we seek the feedback equilibrium strategies which are subgame-perfect.

Throughout the paper, we make the following assumptions regarding the parameter values:

Assumption 1 (A1). $a_i - bc_1 > 0$, for $i = 1, 2$.

Assumption 2 (A2). $c_1 - a_i\gamma > 0$, for $i = 1, 2$.

(A1) ensures that the demand and the production quantities in both periods are positive. (A2) requires that the learning parameter γ is in an appropriate range so that the second-period production cost remains positive in the equilibrium. Note that with these two assumptions, one can easily verify that $0 < b\gamma < 1$.

4.4 Inventory Not Allowed

In this section, we consider the case in which inventories are not allowed. This could happen when the retailer does not have physical capacity (for example, warehouse or storage rooms). Both the manufacturer and the retailer know that no inventory will be carried before the game starts and they make decisions accordingly. The manufacturer produces exactly what the retailer orders in each period and there is no capacity constraint (i.e., the manufacturer's capacity is infinite). We derive the equilibrium pricing and ordering polices for the centralized channel and then the decentralized channel. We compare the results from centralized channel with those from the decentralized channel. We are interested in examining how the learning curve affects the double marginalization and determining whether revenue sharing contracts can coordinate the dynamic decentralized channel. When coordination is achieved, how do the revenue sharing rates affect the coordinating dynamic wholesale price terms and how do they affect the the splitting of profit between the two parties.

4.4.1 The Centralized Channel

We begin the analysis by considering a centralized (integrated) channel in which the manufacturer and the retailer are under the same ownership. This centralized channel serves as the benchmark for the decentralized channel in which the retailer is independent from the manufacturer as we will explore subsequently. Let π be the centralized channel's total profit over the two periods, and $\pi_2(x)$ be the channel's profit in Period 2, given that the learning rate is x (Recall that the learning rate x is stochastic). In the case of deterministic demand and no inventory allowed, there is an one-to-one mapping between the retail price p_i and the order quantity q_i. We select the retail price p_i to be the decision variables for the centralized channel (for the retailer in the decentralized channel). The optimal order quantity q_i can be determined accordingly. The integrated channel solves the following dynamic program:

$$\pi^* = \max_{p_1} \left\{ \pi = (p_1 - c_1)(a_1 - bp_1) + E[\pi_2^*(x)] \right\}, \tag{4.1}$$

$$\pi_2^*(x) = \max_{p_2} \left\{ \pi_2(x) = (p_2 - c_2(x))(a_2 - bp_2) \right\}, \tag{4.2}$$

where $c_2(x) = c_1 - q_1\gamma x = c_1 - (a_1 - bp_1)\gamma x$ and $E[\pi_2^*(x)] = \int_0^1 \pi_2^*(x) f(x) dx$. The dynamics in this model come from the cost learning effect.

Proposition 4.4.1 *When inventories are not allowed to be carried, the equilibrium retail prices p_1^* and p_2^*, the production quantities q_1^* and q_2^*, and the second-period production cost c_2^* are summarized in Table 4.1.*

Table 4.1. Equilibrium Prices and Quantities in a Centralized Channel

	Inventory Not Allowed	Inventory Allowed
p_1^*	$\frac{2(a_1+bc_1)-b(a_2-bc_1)\gamma\mu-a_1b^2\gamma^2(\mu^2+\sigma^2)}{b(4-b^2\gamma^2(\mu^2+\sigma^2))}$	$\frac{4\mu(a_1+bc_1)-b\gamma((3a_1+a_2)\mu^2+(a_1+bc_1)\sigma^2)-bh(2\mu-b\gamma(\mu^2+\sigma^2))}{2b[4\mu-b\gamma(2\mu^2+\sigma^2)]}$
q_1^*	$\frac{2(a_1-bc_1)+b\gamma\mu(a_2-bc_1)}{4-b^2\gamma^2(\mu^2+\sigma^2)}$	$\frac{\gamma\mu(a_1+a_2-2bc_1)-(2-b\gamma\mu)h}{\gamma[4\mu-b\gamma(2\mu^2+\sigma^2)]}$
p_2^*	$\frac{(4-b^2\gamma^2\sigma^2)(a_2+bc_1)-2b\gamma\mu(a_1-bc_1)-2a_2b^2\gamma^2\mu^2}{2b(4-b^2\gamma^2(\mu^2+\sigma^2))}$	$\frac{4\mu(a_2+bc_1)-b\gamma\left[(a_1+3a_2)\mu^2+(a_2+bc_1)\sigma^2\right]+b\mu h(2-b\gamma\mu)}{2b[4\mu-b\gamma(2\mu^2+\sigma^2)]}$
q_2^*	$\frac{(4-b^2\gamma^2\sigma^2)(a_2-bc_1)+2b\gamma\mu(a_1-bc_1)}{2(4-b^2\gamma^2(\mu^2+\sigma^2))}$	$\frac{\gamma(2\mu-b\gamma\sigma^2)(a_1+a_2-2bc_1)+(4-2b\gamma\mu-b^2\gamma^2\sigma^2)h}{2\gamma[4\mu-b\gamma(2\mu^2+\sigma^2)]}$
c_2^*	$\frac{(4-b^2\gamma^2\sigma^2)c_1-2\gamma\mu(a_1-bc_1)-a_2b\gamma^2\mu^2}{4-b^2\gamma^2(\mu^2+\sigma^2)}$	$\frac{4c_1\mu-(a_1+a_2)\gamma\mu^2-bc_1\gamma\sigma^2+(2-b\gamma\mu)\mu h}{4\mu-b\gamma(2\mu^2+\sigma^2)}$

All proofs appear in the section 4.8. We subsequently compare them with the results from the decentralized channel. Furthermore, when designing the coordinating revenue sharing contracts, we use the above results as the benchmark case.

4.4.2 Decentralized Channel

Now we consider a decentralized channel in which the manufacturer and the retailer are under different ownership so that the two parties make independent decisions. The manufacturer announces the wholesale prices sequentially in each period. The sequence of the events is as follows. In the first period, the manufacturer announces the wholesale price w_1.

The retailer decides the retail price p_1 and the order quantity q_1. The manufacturer produces q_1 at a cost of c_1 per unit. The retail market clears in the first period. Meanwhile, the learning rate and hence the production cost c_2 are resolved at the end of the first period. In the second period, the manufacturer announces the wholesale price w_2. The retailer decides the retail price p_2 and the order quantity q_2. The manufacturer produces q_2 and the retail market clears in the second period.

In the second period, given the realized learning rate x, the manufacturer and the retailer each solves the following problems:

$$\pi_{r2}^*(x) = \max_{p_2} \left[\pi_{r2} = (p_2 - w_2)(a_2 - bp_2)\right], \tag{4.3}$$

$$\pi_{m2}^*(x) = \max_{w_2} \left[\pi_{m2} = (w_2 - c_2(x))\left(a_2 - bp_2(w_2)\right)\right], \tag{4.4}$$

where $c_2(x) = c_1 - \gamma x q_1$ and $p_2(w_2)$ is the retailer's best response retail price in Period 2. In the first period, the retailer and the manufacturer each maximizes the total profit from two periods:

$$\pi_r^* = \max_{p_1} \left\{\pi_r(p_1) = (p_1 - w_1)(a_1 - bp_1) + \int_0^1 \pi_{r2}^*(x) f(x) dx\right\},$$

$$\pi_m^* = \max_{w_1} \left\{\pi_m(w_1) = (w_1 - c_1)(a_1 - bp_1) + \int_0^1 \pi_{m2}^*(x) f(x) dx\right\},$$

where $\pi_{r2}(x)$ and $\pi_{m2}(x)$ are given by (4.3) and (4.4), respectively. In contrast to a forward-looking objective, a myopic policy is to maximize the current period profit. Mazzola and McCardle (1997) analyze both the myopic and optimal (forward-looking) production policies in the presence of learning curve effect.

We derive the unique subgame-perfect feedback equilibrium prices and order quantities in each period and the second-period production cost in the following proposition.

Proposition 4.4.2 *When inventories are not allowed, the equilibrium retail prices, the production quantities, and the second-period production cost are listed in Table 4.2.*

Under assumptions (A1) and (A2), it can be easily verified that $w_1^* > 0$, $w_2^* > 0$, $p_1^* > 0$, $p_2^* > 0$, $q_1^* > 0$, $q_2^* > 0$, and $c_2^* > 0$.

Table 4.2. Equilibrium Results in a Decentralized Channel

	Inventory Not Allowed
Retail price p_1^*	$\frac{8(3a_1+bc_1)-b\gamma\left[3\mu(a_2-bc_1)+4a_1b\gamma(\mu^2+\sigma^2)\right]}{4b[8-b^2\gamma^2(\mu^2+\sigma^2)]}$
Quantity q_1^*	$\frac{8(a_1-bc_1)+3b\gamma\mu(a_2-bc_1)}{[8-b^2\gamma^2(\mu^2+\sigma^2)]}$
Wholesale price w_1^*	$\frac{1}{32b}\left\{8\left(3a_1+bc_1\right)+b\gamma\mu(a_2-bc_1)-\frac{8[8(a_1-bc_1)+3b\gamma\mu(a_2-bc_1)]}{8-b^2\gamma^2(\mu^2+\sigma^2)}\right\}$
Retail price p_2^*	$\frac{1}{16b}\left\{4\left(3a_2+bc_1\right)-\frac{b\gamma\mu[8(a_1-bc_1)+3b\gamma\mu(a_2-bc_1)]}{8-b^2\gamma^2(\mu^2+\sigma^2)}\right\}$
Order quantity q_2^*	$\frac{(32-b^2\gamma^2(\mu^2+4\sigma^2))(a_2-bc_1)+8b\gamma\mu(a_1-bc_1)}{16[8-b^2\gamma^2(\mu^2+\sigma^2)]}$
Wholesale price w_2^*	$\frac{1}{8b}\left\{4\left(a_2+bc_1\right)-\frac{b\gamma\mu[8(a_1-bc_1)+3b\gamma\mu(a_2-bc_1)]}{8-b^2\gamma^2(\mu^2+\sigma^2)}\right\}$
Production cost c_2^*	$\frac{c_1\left[32+b\gamma(\mu(8-b\gamma\mu)-4b\gamma\sigma^2)\right]-\gamma\mu(8a_1+3a_2b\gamma\mu)}{4[8-b^2\gamma^2(\mu^2+\sigma^2)]}$

Proposition 4.4.3 *(i)* $p_1^{D*} > p_1^{C*}, q_1^{D*} < q_1^{C*}, p_2^{D*} > p_2^{C*}, q_2^{D*} < q_2^{C*}, c_2^{D*} > c_2^{C*}, \pi_c^{D*} < \pi_c^{C*}$. *(ii) The comparative statics with regard to* μ *and* σ *are:* $\partial(p_1^{D*} - p_1^{C*})/\partial\mu > 0, \partial(p_1^{D*} - p_1^{C*})/\partial\sigma > 0, \partial(q_1^{C*} - q_1^{D*})/\partial\mu > 0, \partial(q_1^{C*} - q_1^{D*})/\partial\sigma > 0, \partial(p_2^{D*} - p_2^{C*})/\partial\mu > 0, \partial(p_2^{D*} - p_2^{C*})/\partial\sigma > 0, \partial(q_2^{C*} - q_2^{D*})/\partial\mu > 0, \partial(q_2^{C*} - q_2^{D*})/\partial\sigma > 0, \partial(c_2^{D*} - c_2^{C*})/\partial\mu > 0, \partial(c_2^{D*} - c_2^{C*})/\partial\sigma > 0, \partial(\pi_c^{C*} - \pi_c^{D*})/\partial\mu > 0, \partial(\pi_c^{C*} - \pi_c^{D*})/\partial\sigma > 0$.

Not surprisingly, the retailer charges higher prices in each period in the decentralized channel than those in the centralized channel due to the well-known double marginalization effect. Accordingly, the production/order quantities are lower in the decentralized channel. Suboptimal production in the first period leads to insufficient learning in the first period. In other words, cost reduction opportunities due to volume learning cannot be fully utilized in the decentralized channel. The efficiency loss, i.e., the profit difference $(E[\pi_c^{C*}] - E[\pi_c^{D*}])$, is increasing in the learning rate mean μ and standard deviation σ. The faster the manufacturer learns (larger μ) or the more unstable the learning impact becomes(larger σ), the more severe is double marginalization. Proposition 4.4.3 highlights the importance of channel coordination in the dynamic decentralized supply chain when learning curve effect is present as learning can magnify the efficiency loss. We next explore whether the revenue sharing contracts, a commonly used coordination mechanism, can coordinate the decentralized channel and restore supply chain efficiency.

4.4.3 Revenue-sharing Contract

Suppose the manufacturer and the retailer agree to use a revenue sharing contract at the beginning of the horizon. Under such a contract, the manufacturer sequentially proposes two pairs of wholesale price and revenue sharing rate $\{w_1, \phi_1\}$ and $\{w_2, \phi_2\}$, $0 \leqslant \phi_1, \phi_2 \leqslant 1$, to the retailer, where ϕ_i is the retailer's portion of the revenue in Period $i = 1, 2$. The retailer pays a wholesale price of w_i per unit in period i. With a revenue-sharing term $\{w_2, \phi_2\}$, the retailer's and the manufacturer's second-period profit functions are:

$$\pi_{r2}(x) = (\phi_2 p_2 - w_2(x))(a_2 - bp_2) \tag{4.5}$$

$$\pi_{m2}(x) = \left[(1-\phi_2)p_2 + w_2 - c_2(x))\right](a_2 - bp_2). \tag{4.6}$$

The first-period profit functions to the manufacturer and the retailer are given by:

$$\pi_r = (\phi_1 p_1 - w_1)(a_1 - bp_1) + \int_0^1 \pi_{r2}^*(x) f(x) dx,$$

$$\pi_m = \left[(1-\phi_1)p_1 + w_1 - c_1\right](a_1 - bp_1) + \int_0^1 \pi_{m2}^*(x) f(x) dx.$$

where $\pi_{r2}(c)$ and $\pi_{m2}(x)$ are given by (4.5) and (4.6), respectively. We use the centralized channel in Section 4.1 as the benchmark case. We refer to the scenario as *full channel coordination* when the total profit in a decentralized channel is equal to that in the integrated channel. Theorem 4.4.4 shows that properly designed revenue sharing contracts can fully coordinate the two-period dynamic supply chain.

Theorem 4.4.4 *For $\phi_1, \phi_2 \in [0, 1]$, let the manufacturer set the wholesale prices as*

$$w_1^* = \phi_1 c_1 + \frac{(\phi_2 - \phi_1)\gamma\left[2\mu\left(a_2 - bc_1\right) + (a_1 - bc_1)b\gamma(\mu^2 + \sigma^2)\right]}{4 - b^2\gamma^2(\mu^2 + \sigma^2)}$$

and $w_2^*(x) = \phi_2 c_2^*(x) = \phi_2(c_1 - x\gamma q_1^*)$ for any given learning rate x, where q_1^*, specified in Proposition 4.4.1, is the equilibrium first-period production in the centralized channel when inventories are not allowed. Then, the revenue sharing contract coordinates the supply chain. Moreover, if $\phi_1 = \phi_2 = \phi$, then $w_1^* = \phi c_1$ and $w_2^*(x) = \phi c_2^*(x)$.

Without inventory, the retailer's portions of revenue, sharing rates ϕ_1 and ϕ_2, can be arbitrarily selected and they do not need to be the same in two periods. This implies that the coordinating revenue sharing contract arbitrarily splits the channel profit between the manufacturer and the retailer, a similar observation made by Cachon and Lariviere (2005) in a static supply chain with a single revenue sharing rate. Note that the coordinating policies are obtained when the behaviors parameters of the independent retailer are equated with those of the centralized channel. Under a two-period revenue sharing contract, the manufacturer has four degrees of freedom in specifying the coordinating contract (the two wholesale prices and the two two revenue sharing rates), which are sufficient to equate the two parameters that govern the retailer's behavior (the retail prices in Periods 1 and 2). Hence, there are two additional degrees of freedom to coordinate the two-period channel.

One implication of Theorem 1 is that the two parties could negotiate over two revenue sharing rates and they have additional flexibility to balance the profit from the first and second periods. One may conjecture that in coordination, as the manufacturer offers more favorable sharing rates (i.e., as ϕ_1 and ϕ_2 increase), he also increases the wholesale prices w_1^* and w_2^*. In other words, does the retailer always have to pay higher wholesale prices in exchange for higher shares of the revenue? How do the revenue sharing rates affect the splitting of the profit between the channel members? Next proposition provides the answers to these questions.

Proposition 4.4.5 $\partial w_1^*/\partial\phi_1 > 0, \partial w_1^*/\partial\phi_2 > 0, \partial w_2^*/\partial\phi_1 = 0, \partial w_2^*/\partial\phi_2 > 0, \partial\pi_r^*/\partial\phi_1 = -\partial\pi_m^*/\partial\phi_1 > 0$. *If* $a_2 \geqslant \Phi(c_1) \equiv bc_1 + \frac{2bc_1(1-b\gamma)\sqrt{\mu^2+\sigma^2}}{4-b^2\gamma^2(\mu^2+\sigma^2)-b^2\gamma^2\mu\sqrt{\mu^2+\sigma^2}}$, *then* $\partial\pi_r^*/\partial\phi_2 = -\partial\pi_m^*/\partial\phi_2 > 0$. *If* $a_2 < \Phi(c_1)$ *and* $a_1 \leqslant \Psi(a_2) \equiv bc_1 + \frac{(a_2-bc_1)\left[4-b^2\gamma^2(\mu^2+\sigma^2)-b^2\gamma^2\mu\sqrt{\mu^2+\sigma^2}\right]}{2b\gamma\sqrt{\mu^2+\sigma^2}}$, *then* $\partial\pi_r^*/\partial\phi_2 = -\partial\pi_m^*/\partial\phi_2 > 0$. *If* $a_1 > \Psi(a_2)$, $\partial\pi_r^*/\partial\phi_2 = -\partial\pi_m^*/\partial\phi_2 < 0$.

The coordinating first-period wholesale price is increasing in the revenue sharing rates in each period which reflects the balance between a revenue portion and the wholesale price. The second-period wholesale price is independent of ϕ_1 while it is increasing in the second-period revenue sharing rate ϕ_2. The impact of first-period revenue sharing rate on the channel member's profit is implicit: the manufacturer's (retailer's) profit is decreasing (increasing) in ϕ_1. The reason is that the manufacturer increases the first-period wholesale price in the association of a higher portion of revenue for the retailer, but he is unable to increase the second-period wholesale price. The manufacturer is worse off with a higher ϕ_1.

The impact of second-period revenue sharing rate on π_m^* and π_r^* is undetermined. One may conjecture that the retailer is better off when she receives a higher portion of the revenue in the second period. We find that this intuition holds in the *regular* case where the second-period market size is medium or large and the first period market size is not very large. However, in an *irregular* case when the first-period market size is sufficiently large as shown in Figure 4.1, the retailer's profit decreases in the second-period revenue sharing rate. This happens because with an increase in ϕ_2, the manufacturer increases w_2 as well as w_1. In addition to that, when a_1 is very large, the retailer's benefit of getting a large portion of second-period revenue is overly mitigated by the prices she has to pay to the manufacturer. So she actually can be better off with a lower ϕ_2. This phenomenon happens only when the learning effect exists, i.e., $\gamma, \mu > 0$. If there is no learning effect, the retailer's profit always increases in the second-period revenue sharing rate.

In a dynamic setting, the channel members need to be aware of the intertemporal effects of the revenue sharing rates.

4.5 Inventory Allowed

In this section, we allow the retailer to carry over inventory so that the retailer can order an amount that is larger than the demand in the first period, i.e., $q_1 \geqslant D_1 = a_1 - bp_1$. The inventory is carried to satisfy the second-period demand with an inventory holding cost h per unit. Without loss of generality, we assume that the inventory level at the beginning of Period 1 is zero, i.e., $I_1 = 0$. The retailer orders q_1 from the manufacturer in Period 1. Backlog is not allowed, i.e., the retailer cannot sell more than what the manufacturer

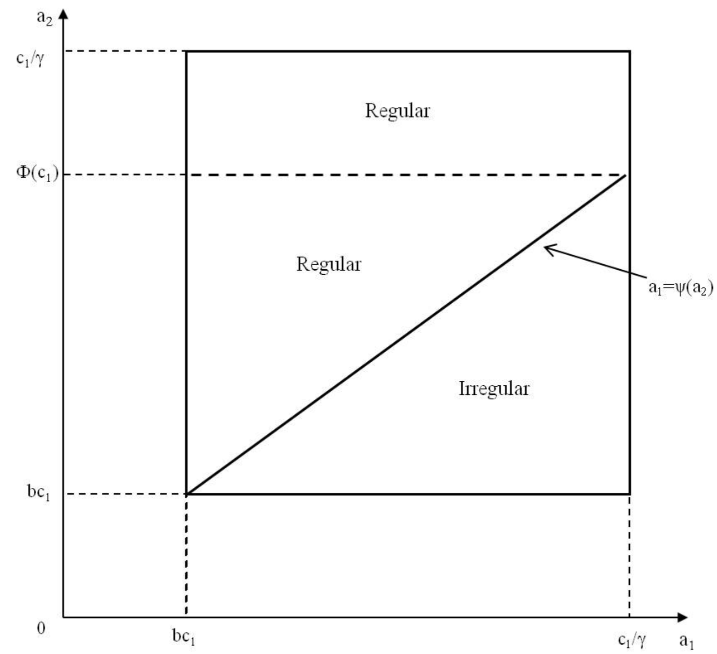


Figure 4.1. Impact of Second-Period Revenue Sharing Rate ϕ_2

produces in each period. If the available units are more than the demand in Period 1, i.e., $q_1 > D_1$, then the retailer has an inventory $I_2 = q_1 - D_1$ on hand at the beginning of the second period. Otherwise, the retailer sells to satisfy the first period demand and carries no inventory to Period 2. We are particularly interested in determining when it is optimal for the retailer to carry over inventory. When the inventory is carried over, we are interested in how the parameters affect the optimal inventory level. We investigate in what terms the revenue sharing contracts can coordinate the channel with inventory carry-over.

4.5.1 The Centralized Channel

In Period 1, the centralized channel sets the retail price p_1 and decides on the production quantity q_1. In Period 2, given the initial stock I_2 and the realized learning rate x, the centralized channel chooses the second-period retail price p_2 and decides on the second-period production quantity q_2. Let $[y]^+ = \max\{y, 0\}$. The integrated channel's dynamic optimization problems in two periods are given by:

$$\pi^* = \max_{p_1>0,q_1\geqslant 0}\left\{\pi(x) = p_1(a_1-bp_1) - c_1q_1 + \int_0^1 \pi_2^*(x)f(x)dx\right\} \tag{4.7}$$

$$\pi_2^*(x) = \max_{p_2>0,q_2\geqslant 0}\left\{\pi_2(x) = p_2(a_2-bp_2) - c_2(x)q_2 - h[I_2]^+\right\} \tag{4.8}$$

where $c_2 = c_1 - \gamma x q_1$ and $I_2 = q_1 - (a_1 - bp_1)$.

Lemma 4.5.1 presents the *inventory threshold*, the point at which the centralized channel moves from $I_2^* = 0$ to $I_2^* > 0$.

Lemma 4.5.1 *The inventory threshold in the centralized channel*

$$S_{cn} = -h\left[4 - b^2\gamma^2(\mu^2+\sigma^2)\right] + \gamma\left[\mu(a_2-a_1)(2-b\gamma\mu) + b\gamma\sigma^2(a_1-bc_1)\right]. \tag{4.9}$$

(a) If $S_{cn} > 0$, then

$$I_2^* = \frac{S_{cn}}{2\gamma\left[4\mu - b\gamma(2\mu^2+\sigma^2)\right]} > 0.$$

The corresponding equilibrium retail prices and production quantities in each period and the second-period production cost are summarized in the third column of Table 4.1.

(b) If $S_{cn} < 0$, then $I_2^* = 0$, and the problem reduces to that when inventories are not allowed.

We have a few observations. First, when the learning curve effect is absent, i.e., $\gamma = 0$, the centralized channel does not carry inventory. Second, when the learning effect exists, inventories may be carried. Third, the threshold S_{cn} is increasing in σ. This implies that the centralized channel is more likely to carry inventories when the efficiency of learning is less predictable (a high σ). When the market sizes are symmetric, i.e., $a_1 = a_2$, the inventories are not carried.

4.5.2 Decentralized Channel

The manufacturer sets the wholesale price in Period 2. Given that its initial stock (before purchase) in Period 2 is I_2 and the announced wholesale price, the retailer decides on retail price p_2 and order quantity q_2. The retailer and the manufacturer's second period problems are:

$$\pi^*_{r2}(x) = \max_{p_2>0, q_2\geqslant 0} \{\pi_{r2}(x) = p_2(a_2 - bp_2) - w_2 q_2 - h[I_2]^+\} \tag{4.10}$$

$$\pi^*_{m2}(x) = \max_{w_2\geqslant 0} \{\pi_{m2} = (w_2 - c_2(x))\left(a_2 - bp_2 - [I_2]^+\right)\} \tag{4.11}$$

where $c_2 = c_1 - q_1\gamma x$, $q_2 = a_2 - bp_2 - [I_2]^+$, and $I_2 = q_1 - (a_1 - bp_1)$. Since the second-period demand is deterministic and it is the ending selling period, there is an one-to-one mapping between the retail price and the demand. Again, we choose its retail price as the decision variable. In the first period, the retailer chooses the order quantity and retail price while the manufacturer decides on the wholesale price:

$$\pi^*_r = \max_{p_1\geqslant 0, q_1\geqslant 0} \pi_r(p_1, q_1) = p_1(a_1 - bp_1) - w_1 q_1 + \int_0^1 \pi^*_{r2}(x) f(x) dx$$

$$\pi^*_m = \max_{w_1\geqslant 0} \pi_m(w_1) = (w_1 - c_1)q_1 + \int_0^1 \pi^*_{m2}(x) f(x) dx$$

where $\pi_{r2}(x)$ and $\pi_{m2}(x)$ are given by (4.10) and (4.11), respectively. Similar to the centralized channel, Lemma 4.5.2 presents the *inventory threshold* for the decentralized channel, the point at which the retailer moves from $I^*_2 = 0$ to $I^*_2 > 0$.

Lemma 4.5.2 *The inventory threshold in the decentralized channel is*

$$\begin{aligned} S_d = \; & -bh\left[160 + 32b\gamma\mu - b^2\gamma^2\left(31\mu^2 + 28\sigma^2\right)\right] \\ & + (a_2 - bc_1)\left[40 + 28b\gamma\mu + b^2\gamma^2(2\mu^2 + 7\sigma^2)\right] \\ & + (a_2 - a_1)\left[48 + 46b\gamma\mu - b^2\gamma^2\left(31\mu^2 + 28\sigma^2\right)\right] \end{aligned}$$

(a) If $S_d > 0$, the retailer carries inventory

$$I^*_2 = \frac{S_d}{2\left[136 + 120b\gamma\mu - b^2\gamma^2(60\mu^2 + 49\sigma^2)\right]} > 0.$$

The corresponding equilibrium retail and wholesale prices, production quantities in each period and the second-period production cost are summarized in the Appendix.
(b) If $S_d < 0$, then $I_2^* = 0$. The equilibrium results are the same as those when inventories are not allowed.

Recall that in the centralized channel, when $\gamma = 0$, inventories are not carried even though they are allowed. In contrast, in the decentralized channel, the retailer may carry inventory even when there is no learning curve effect[3]. Under such occasions, inventories arise not for the classical reasons but for *strategic* considerations, a result obtained by Anand et al. (2008) for a special case of our problem with $\gamma = 0$ and $a_1 = a_2$. Strategic inventories are used by the retailer to curtail the second-period monopoly power of the supplier. When learning effect does exist, we conjecture that these insights continue to hold. We shall investigate the values of strategic inventories to the channel members in the context of learning effect.

4.5.3 Revenue Sharing Contract

It is more challenging to achieve channel coordination (first-best scenario) in a dynamic setting as the retailer has more decisions to make. Anand et al. (2008) show that the two part-tariff contract (a contract with a fixed fee and a wholesale price) cannot achieve the first best solution because the buyer carries inventory. We now consider the revenue sharing contracts that can coordinate the dynamic supply chain with learning effect and when the retailer is allowed to carry inventory.

The retailer's problems in Period 1 and Period 2 are given by:

$$\pi_r^* = \max_{p_1, q_1 \geqslant 0} \left\{ \pi_r = \phi_1 p_1 (a_1 - b_1 p_1) - w_1 q_1 + \int_0^1 \pi_{r2}^*(x) f(x) dx \right\}$$
$$\pi_{r2}^*(x) = \max_{p_2, q_2 \geqslant 0} \left\{ \pi_{r2}(x) = \phi_2 p_2 (a_2 - b_2 p_2) - w_2(x) q_2 - h[I_2]^+ \right\}$$

where $I_2 = q_2 - (a_2 - b_2 p_2)$.

[3] When $\gamma = 0$ and $88a_2 - 48a_1 - 40bc_1 > 160bh$, the retailer carries inventory in the equilibrium.

The manufacturer's profit functions are given by:

$$\pi_m = (1-\phi_1)p_1(a_1 - b_1 p_1) + (w_1 - c_1)q_1 + \int_0^1 \pi^*_{m2}(x) f(x)dx$$
$$\pi_{m2}(x) = (1-\phi_2)p_2(a_2 - b_2 p_2) + (w_2 - c_2(x))q_2$$

Recall that S_{cn} denotes the inventory threshold in a centralized channel given by (4.9). Define $A = -\gamma\mu^2(a_1 + a_2) + c_1(4\mu - b\gamma\sigma^2) > 0$.

Theorem 4.5.3 *(i) If $S_{cn} > 0$, consider the following revenue sharing contract set $(\phi_1, \phi_2, w_1^*, w_2^*)$: set $w_1^* = c_1\phi_2 - h(1-\phi_2)$, where $\phi_1 \in [0, 1]$, $\phi_2 \in \left[\frac{h(4\mu - b\gamma(2\mu^2+\sigma^2))}{A+(2-b\gamma\mu)\mu h}, 1\right]$ and ϕ_1 and ϕ_2 satisfy the following relationship:*

$$\phi_1 = \phi_2 - \frac{h(1-\phi_2)\left[4\mu - b\gamma(2\mu^2 + \sigma^2)\right]}{A - h\left[2\mu - b\gamma(\mu^2 + \sigma^2)\right]};$$

set $w_2(x) = \phi_2 c_2^*(x) = \phi_2(c_1 - x\gamma q_1^*)$, where q_1^* is the equilibrium first period production quantity in a centralized channel when inventories are allowed and is given by Lemma 4.5.1.

(a)The above policy of $(\phi_1, \phi_2, w_1^*, w_2^*)$ achieves channel coordination.

(b) $\phi_1 \leqslant \phi_2$. When $h = 0$, $\phi_1 = \phi_2$.

(ii) If $S_{cn} \leqslant 0$, then no inventory is carried and the coordinating contracts are the same as those when inventories are allowed which are described in Theorem 4.4.4.

Theorem 4.5.3 shows that revenue sharing contracts can coordinate the decentralized channel when inventory is carried. The manufacturer has four degrees of freedom to specify the revenue sharing contracts. However, when inventories are allowed and are actually carried in equilibrium, he has to equate the three parameters that govern the retailer's behavior (the retail prices in Periods 1 and 2 and the initial inventory level in Period 2). Thus, the manufacturer has one degree of freedom to choose the contract parameter and he no longer has the flexibility to pick up both rates: the two rates are interdependent. In coordination, the second period revenue sharing rate is required to be greater than or equal to the first-period revenue sharing rate.

For a special case of zero holing cost $h = 0$, the two rates are the same: $\phi_1 = \phi_2$.

Table 4.3. Impact of Holding Cost $(a_1 = a_2 = a, \gamma = 0)$

	No Inventory	Inventory
(w_1, w_2)	$\left\{\frac{a+bc_1}{2b}, \frac{a+bc_1}{2b}\right\}$	$\left\{\frac{9a+8bc_1-2bh}{17b}, \frac{6a+11bc_1+10bh}{17b}\right\}$
(p_1, p_2)	$\left\{\frac{3a+bc_1}{4b}, \frac{3a+bc_1}{4b}\right\}$	$\left\{\frac{13a+4bc_1-bh}{17b}, \frac{23a+11bc_1+10bh}{34b}\right\}$
(q_1, q_2)	$\left\{\frac{a-bc_1}{4}, \frac{a-bc_1}{4}\right\}$	$\left\{\frac{13a-13bc_1-18bh}{34}, \frac{3a-3bc_1+5bh}{17}\right\}$
(D_1, D_2)	$\left\{\frac{a-bc_1}{4}, \frac{a-bc_1}{4}\right\}$	$\left\{\frac{(4a+b(-4c_1+h)}{17}, \frac{11a-11bc_1-10bh}{34}\right\}$
I_2	0	$\frac{5(a-b(c_1+4h))}{34}$
π_r	$\frac{(a-bc_1)^2}{8b}$	$\frac{155(a-bc_1)^2+118b(-a+bc_1)h+304b^2h^2}{1156b}$
π_m	$\frac{(a-bc_1)^2}{4b}$	$\frac{9(a-bc_1)^2+4b(-a+bc_1)h+8b^2h^2}{34b}$

4.6 Value of Strategic Inventory

In the previous sections, we derive the results for the cases when inventories are allowed and when they are not allowed to carry. We use superscripts N and I to denote the cases when inventories are not allowed and when inventories are allowed, respectively. We derive the preferences of manufacturer, retailer, and supply chain with regard to inventory versus no inventory by comparing the profits of each channel member under these two cases. We are interested in identifying the value of strategic inventories to the channel members.

4.6.1 Impact of Holding Cost h

We explore how the holding cost h affects the retailer's, the manufacturer's, and the supply chain's profits in the two cases. In order to focus on the effects of inventory holding cost, we consider a special case without learning effect and with equal market potentials in two periods. We set $\gamma = 0$ and $a_1 = a_2 = a$ to get the results summarized in Table 4.3. This problem reduces to the one studied by Anand et al. (2008), which is the first to identify the *strategic* inventories in a vertical supply chain. Note that with $\gamma = 0$, the centralized channel does not carry inventory. So in the decentralized channel, whenever the retailer carries inventory, it is carried for pure strategic considerations.

Proposition 4.6.1 *If there is no learning effect and the market potential is symmetric in* *two periods,* $\pi_m^{I*} \geqslant \pi_m^{N*}$. *The retailer's profits comparisons are*

$$\pi_r^{I*} - \pi_r^{N*} \begin{cases} > 0, & \textit{if } 0 \leqslant h < \frac{21(a-bc_1)}{152b}, \\ < 0, & \textit{if } \frac{21(a-bc_1)}{152b} < h < \frac{a-bc_1}{4b}, \\ = 0, & \textit{if } h \geqslant \frac{a-bc_1}{4b}. \end{cases}$$

The supply chain's profits comparisons are:

$$\pi_c^{I*} - \pi_c^{N*} \begin{cases} > 0, & \textit{if } 0 \leqslant h < \dfrac{55(a-bc_1)}{288b}, \\ < 0, & \textit{if } \dfrac{55(a-bc_1)}{288b} < h < \dfrac{a-bc_1}{4b}, \\ = 0, & \textit{if } h \geqslant \dfrac{a-bc_1}{4b}. \end{cases}$$

In a decentralized channel when $\gamma = 0$ and $a_1 = a_2$, the retailer carries strategic inventory when inventory is not too costly to carry, i.e., $h < \frac{a-bc_1}{4b}$; otherwise, the retailer does not carry inventory. When inventories are carried, the manufacturer increases his first-period wholesale price ($w_1^{N*} < w_1^{I*}$). By carrying strategic inventories, the retailer forces the manufacturer to price only for the retailer's residual demand in the second period (Anand et al. 2008). The retailer's strategic inventories are used to mitigate the manufacturer's monopoly power in the second period. Let $w_{avg}^{I*} = (w_1^{I*} + w_2^{I*})/2$ and $w_{avg}^{N*} = (w_1^{N*} + w_2^{N*})/2$ denote the average wholesale prices when inventories are allowed and when inventories are not allowed, respectively. From Table 4.3, we have $w_{avg}^{I*} < w_{avg}^{N*}$: the overall effect is that the retailer forces the manufacturer to reduce the wholesale price when she carries strategic inventories.

Interestingly, the manufacturer benefits when the retailer carries strategic inventories. However, the retailer may be better off or worse off when she carries inventory. Specifically, for a low inventory holding cost h ($0 < h < \frac{21(a-bc_1)}{152b}$), the retailer carries inventory and she is better off doing so than when inventories are not allowed at all.

For a medium value of h, the retailer carries inventory in equilibrium but she is worse off with inventory. Two questions arise here. Why is the retailer worse off with inventory? Why does the retailer carry inventory if she is worse off? For the first question, the retailer's preferences between inventory versus no inventory reflect a trade-off between the retailer's power and inventory holding cost. Although the retailer gains more power with strategic inventory, the power comes with the inventory holding cost which the retailer incurs solely. For a low h, the retailer's power outweighs the holding cost so she is better off carrying inventory. For a medium or high h, the holding cost is so high that inventory carryover option hurts the retailer. For very high h, the retailer does not carry inventory as it is too costly. For the second question, when inventories are allowed to carry, the manufacturer sets the wholesale prices in each period in such a way that the retailer has to carry inventory.

Now let's look at the supply chain's preference. For a low value of h, when inventory is carried, the retailer's total order quantity from two periods is higher, i.e., $q_1^{I*} + q_2^{I*} > q_1^{N*} + q_2^{N*}$. Let $p_{avg}^{I*} = (p_1^{I*} + p_2^{I*})/2$ and $p_{avg}^{N*} = (p_1^{N*} + p_2^{N*})/2$ denote the average retail prices when inventories are and inventories are not allowed. From Table 4.3, we have $p_{avg}^{I*} < p_{avg}^{N*}$ for $0 < h < \frac{a-bc_1}{4h}$. Therefore, the strategic inventories mitigate the double marginalization problem. The improvement in the profit benefits both the manufacturer and retailer therefore the supply chain's profit is higher when inventory is carried. As the manufacturer controls the wholesale prices thus control the strategic inventories, with symmetric markets, he is always better off with low and medium inventory cost. However, for very high inventory holding cost, he is unable to extract the profit as the wholesale prices are not sufficient to recoup the loss from the low order quantity.

4.6.2 Impact of Learning Efficiency Mean

In this subsection, we study the impact of learning rate parameter γ on the value of inventory carryover. We consider a special case of $h = 0$ and $a_1 = a_2$. To focus on the impact the learning efficiency mean, we assume $\sigma = 0$ and $\mu = 1$ so that the problem reduces to a deterministic learning, i.e., $c_2 = c_1 - q_1\gamma$. Define $G(b\gamma)$ as a function of $b\gamma$ given in the Appendix. For this special case, in a decentralized channel, the retailer carries inventory in equilibrium when she is allowed to do so but the centralized channel does not carry inventory (see Equation 4.9).

Table 4.4. Impact of Learning Efficiency ($a_1 = a_2 = a, h = \sigma = 0, \mu = 1$)

	Inventory not Allowed	Inventory Allowed
w_1	$\frac{24a+8bc_1+b(a-bc_1)\gamma}{32b} - \frac{8(a-bc_1)(8+3b\gamma)}{32b(8-b^2\gamma^2)}$	$\left\{ \frac{-bc_1(32+b\gamma(26+b\gamma(-10+b\gamma)))}{2b(-34+15b\gamma(-2+b\gamma))} + \frac{a(-36+b\gamma(-34+b\gamma(20+b\gamma)))}{2b(-34+15b\gamma(-2+b\gamma))} \right\}$
w_2	$\frac{bc_1(32+b\gamma(8-b\gamma))+a(32-b\gamma(8+7b\gamma))}{8b(8-b^2\gamma^2)}$	$\frac{bc_1(-44+b\gamma(-50+3b\gamma))+a(-24+b\gamma(-10+27b\gamma))}{2b(-34+15b\gamma(-2+b\gamma))}$
(p_1, p_2)	$\left\{ \frac{-bc_1(8+3b\gamma)+a(-24+b\gamma(3+4b\gamma))}{4b(-8+b^2\gamma^2)}, \frac{bc_1(-32+b\gamma(-8+b\gamma))+a(-96+b\gamma(8+15b\gamma))}{16b(-8+b^2\gamma^2)} \right\}$	$\left\{ \frac{bc_1(32+b\gamma(32-b\gamma))+a(104+b\gamma(88-59b\gamma))}{4b[34+15b\gamma(20-b\gamma)]}, \frac{bc_1(44+b\gamma(50-3b\gamma))+a(-92+b\gamma(70-57b\gamma))}{4b[34+15b\gamma(20-b\gamma)]} \right\}$
(q_1, q_2)	$\left\{ \frac{(a-bc_1)(8+3b\gamma)}{4(8-b^2\gamma^2)}, \frac{(a-bc_1)(32+b\gamma(8-b\gamma))}{16(8-b^2\gamma^2)} \right\}$	$\left\{ \frac{(a-bc_1)(26+23b\gamma)}{68+30b\gamma(2-b\gamma)}, \frac{(a-bc_1)(6+b\gamma(9-b\gamma))}{34+15b\gamma(2-b\gamma)} \right\}$
(D_1, D_2)	$\left\{ \frac{(a-bc_1)(8+3b\gamma)}{4(8-b^2\gamma^2)}, \frac{(a-bc_1)(32+b\gamma(8-b\gamma))}{16(8-b^2\gamma^2)} \right\}$	$\left\{ \frac{(a-bc_1)(-32+b\gamma(-32+b\gamma))}{4[34+15b\gamma(2-b\gamma)]}, \frac{(a-bc_1)(88+b\gamma(74-29b\gamma+x(26+23b\gamma)))}{8[34+15b\gamma(2-b\gamma)]} \right\}$
I_2	0	$\frac{(a-bc_1)(20+b\gamma(14+b\gamma))}{4[34+15b\gamma(2-b\gamma)]}$
π_r	$\frac{(a-bc_1)^2(2048+b\gamma(768+b\gamma(-176+b\gamma(-48+7b\gamma))))}{256b(8-b^2\gamma^2)^2}$	$\frac{(a-bc_1)^2\{1240+b\gamma(2384+b\gamma(630+b\gamma(-534+53b\gamma)))\}}{8b[34+15b\gamma(2-b\gamma)]^2}$
π_m	$\frac{(a-bc_1)^2(128+b\gamma(48+b\gamma))}{64b(8-b^2\gamma^2)}$	$\frac{(a-bc_1)^2(36+b\gamma(44+b\gamma))}{4b[34+15b\gamma(2-b\gamma)]}$

Proposition 4.6.2 *When $h = 0$ and $a_1 = a_2 = a$, then $\pi_m^{I*} \geqslant \pi_m^{N*}$, $\pi_c^{I*} \geqslant \pi_c^{N*}$ and*

$$\pi_r^{I*} - \pi_r^{N*} \begin{cases} > 0, & \text{if } 0 \leqslant b\gamma < G_0, \\ \leqslant 0, & \text{if } G_0 \leqslant b\gamma < 1, \end{cases}$$

where $G_0 \approx 0.4579$ is the unique solution to $G(b\gamma) = 0$.

Proposition 4.6.2 shows that both the manufacturer and supply chain are better off when the retailer carries inventory. Interestingly, the retailer is worse off with inventory when the learning process is very efficient (i.e., the manufacturer learns fast), and is better off only when the learning is not significant. The learning favors the manufacturer more than the retailer. The intuition is as follows.

When learning effect is present, inventories are carried not only for strategic considerations but also for operational reasons. The strategic value is mitigated due to the learning effect. The retail and wholesale prices are decreasing across periods: $p_1^{I*} > p_2^{I*}$ and $w_1^{I*} > w_2^{I*}$. The retailer's retail prices decrease in γ in each period[4]. The first-period wholesale price is

[4]We have $\frac{\partial p_1^*}{\partial \gamma} = -\frac{(a-bc_1)(64+b\gamma(446+225b\gamma)}{2(34-15b\gamma(-2+b\gamma))^2} < 0$ and $\frac{\partial p_2^*}{\partial \gamma} = -\frac{(a-bc_1)(95+3b\gamma(93+55b\gamma)}{(34-15b\gamma(-2+b\gamma))^2} < 0$

Table 4.5. Impact of Market Sizes ($h = \gamma = 0$)

	No Inventory	Inventory
Wholesale price (w_1, w_2)	$\left\{\frac{a_1+bc_1}{2b}, \frac{a_2+bc_1}{2b}\right\}$	$\left\{\frac{9(a_1+a_2)+16bc_1}{34b}, \frac{3(a_1+a_2)+11bc_1}{17b}\right\}$
Retail price (p_1, p_2)	$\left\{\frac{3a_1+bc_1}{4b}, \frac{3a_2+bc_1}{4b}\right\}$	$\left\{\frac{43a_1+9a_2+16bc_1}{68b}, \frac{3a_1+20a_2+11bc_1}{34b}\right\}$
Order quantity (q_1, q_2)	$\left\{\frac{a_1-bc_1}{4}, \frac{a_2-bc_1}{4}\right\}$	$\left\{\frac{13(a_1+a_2-2bc_1)}{68}, \frac{3(a_1+a_2-2bc_1)}{17}\right\}$
Demand (D_1, D_2)	$\left\{\frac{a_1-bc_1}{4}, \frac{a_2-bc_1}{4}\right\}$	$\left\{\frac{25a_1-9a_2-16bc_1}{68}, \frac{14a_2-3a_1-11bc_1}{34}\right\}$
Inventory I_2	0	$\frac{11a_2-6a_1-5bc_1}{34}$
Retailer's profit π_r	$\frac{a_1^2+a_2^2-2(a_1+a_2)bc_1+2b^2c_1^2}{16b}$	$\frac{733(a_1-a_2)^2+620(a_1-bc_c)(a_2-bc_1)}{4624b}$
Manufacturer's profit π_m	$\frac{a_1^2+a_2^2-2(a_1+a_2)bc_1+2b^2c_1^2}{8b}$	$\frac{9(a_1+a_2-2bc_1)^2}{136b}$

increasing in γ with a low value of γ and then it is decreasing in γ. The inventory level is increasing in γ initially and then decreasing in γ for a medium and high value[5].

4.6.3 Impact of Market Potential

Recall that with symmetric markets $a_1 = a_2$, the manufacturer is always better off with strategic inventories when learning curve effect is absent. We now investigate how the market sizes a_1 and a_2 affect the retailer's, the manufacturer's, and the supply chain's preferences for inventories. As we focus on the impact of market sizes, we study the special case with no holding cost and no learning curve effect. From Table 4.5, we know that if $a_2 \geqslant \frac{6a_1+5bc_1}{11}$, the retailer carries inventory, and otherwise she does not.

Proposition 4.6.3 *When inventory holding cost is zero and there is no learning curve effect, i.e., $h = 0$ and $\gamma = 0$, we have $\pi_r^{I*} > \pi_r^{N*} > 0$, $\pi_c^{I*} > \pi_c^{N*}$ and*

$$\pi_m^{I*} - \pi_m^{N*} \begin{cases} < 0, & \text{if } a_2 > \frac{9a_1-bc_1+\sqrt{17}(a_1-bc_1)}{8}, \\ > 0, & \text{if } \frac{9a_1-bc_1-\sqrt{17}(a_1-bc_1)}{8} < a_2 < \frac{9a_1-bc_1+\sqrt{17}(a_1-bc_1)}{8}, \\ < 0, & \text{if } \frac{6a_1+5bc_1}{11} < a_2 < \frac{9a_1-bc_1-\sqrt{17}(a_1-bc_1)}{8}, \\ = 0, & \text{if } a_2 \leqslant \frac{6a_1+5bc_1}{11}. \end{cases}$$

For the case of $h = 0$ and $\gamma = 0$, the retailer, as well as the supply chain, is better off when inventories are carried. We find that $p_{avg}^{I*} < p_{avg}^{N*}$. The supply chain is better off as the

[5] We have $\partial I_2^*/\partial\gamma < 0$ for $b\gamma \in [0, 0.1747]$ and $\partial I_2^*/\partial\gamma > 0$ for $b\gamma \in (0.1747, 1)$.

strategic inventories lower the average retail prices and mitigate the double marginalization. This happens partly due to the zero holding cost which encourages the retailer to carry inventory.

However, the manufacturer's profit comparison depends on the relative market sizes. When the second-period market size is very large, the manufacturer's wholesale prices are lower with inventory, i.e., $w_1^{I*} < w_2^{N*}$. Therefore, the manufacturer is better off when the retailer cannot carry inventory. Without learning curve effect, the inventory is carried strategically to mitigate the manufacturer's monopoly power in the second period. The higher the second-period market size, the more inventory the retailer will carry and the larger her negotiation power (control) is with the manufacturer. Hence a_2 favors the retailer, not the manufacturer.

4.7 Conclusion

In a two-period model, we study the learning curve effect in a decentralized supply chain with one manufacturer and one retailer. The manufacturer has opportunities to reduce the (expected) second period per unit production cost due to the first period production experience. We consider a stochastic learning curve, that is the learning rate is random in the first period. We study the implications of learning curve effect on the strategic inventories and dynamic channel coordination. Strategic inventories arise in a two-period decentralized supply chain as the retailer can use them to mitigate the manufacturer's monopoly power in the second period. The outcome of the learning rate is revealed at the end of first period. We study cases when inventories are and are not allowed to carry. For each scenario, we investigate whether a RS contract can coordinate the two-period supply chain. The learning curve effect magnifies the double marginalization effect. We find that RS contract can coordinate the supply chain. However, the coordinating RS contract has different structure. When inventories are allowed and are actually carried in equilibrium, the manufacturer has less flexibility in choosing the revenue sharing rates. As a result, a smaller range of wholesale price contracts can coordinate the supply chain.

One possible extension of our two-period learning curve model is to consider more periods. We conjecture that the strategic inventories still exist as shown by Anand et al. (2008).

It would be interesting to see whether the results would hold with exponential learning. The difficulty with the exponential learning is that it compounds the problem which makes closed-form solutions imposable to attain.

4.8 Proofs

Proof of Proposition 4.4.1.

The fist order condition of $\pi_2(x)$ with regard to p_2 gives us the best response retail price $p_2(x) = \frac{a_2+bc_2(x)}{2b} = \frac{a_2+b[c_1-x\gamma(a_1-bp_1)]}{2b}$. Substitute this best response retail price $p_2(x)$ into $\pi_2(x)$ and the expected second-period profit $E[\pi_2^*(x)]$ is given by:

$$\begin{aligned} E\left[\pi_2^*(x)\right] &= \frac{1}{4b}\int_0^1 \left(a_2 - bc_2(x)\right)^2 f(x)dx \\ &= \frac{1}{4b}\int_0^1 \left[a_2 - b\left(c_1 - x\gamma\left(a_1 - bp_1\right)\right)\right]^2 f(x)dx \\ &= \frac{1}{4b}\int_0^1 \left[\left(a_2 - bc_1\right)^2 + 2b\left(a_2 - bc_1\right)\left(a_1 - bp_1\right)\gamma x + \left(b\gamma x\left(a_1 - bp_1\right)\right)^2\right] f(x)dx \\ &= \frac{1}{4b}\left(a_2 - bc_1\right)^2 + \frac{\gamma\mu}{2}\left(a_2 - bc_1\right)\left(a_1 - bp_1\right) + \frac{1}{4}b\gamma^2\left(a_1 - bp_1\right)^2 E\left[x^2\right] \qquad (4.12) \end{aligned}$$

The channel's first period problem is to

$$\pi^* = \max_{p_1 \geqslant 0}\left\{(p_1 - c_1)(a_1 - bp_1) + E[\pi_2^*(x)]\right\}$$

where $E[\pi_2^*(x)]$ is given by (4.12) which is a function of p_1. We solving the above equation to obtain the equilibrium retail price p_1^* in Proposition 4.4.1. We derive the other equilibrium results q_1^*, $E[c_2^*]$, p_2^*, and p_2^* using the following relationships: $q_1^* = a_1 - bp_1^*$, $E[c_2^*] = c_1 - \gamma\mu q_1^*$, $p_2^* = (a_2 + bE[c_2^*])/(2b)$, and $q_2^* = a_2 - bp_2^*$. ■

Proof of Proposition 4.4.2

The first order condition of π_{r2} with regard to p_2 gives us the best response retail price $p_2(w_2) = \frac{a_2+bw_2}{2b}$. Substitute this response function into and take the first order condition with regard to w_2 gives us $w_2(x) = \frac{a_2+bc_2(x)}{2b}$. The manufacturer and the retailer's profits in the second period are: $\pi^*_{r2}(x) = \frac{(a_2-bc_2(x))^2}{16b}$ and $\pi^*_{m2}(x) = \frac{(a_2-bc_2(x))^2}{8b}$, where $c_2(x) = a_1 - \gamma\mu x q_1$. In Period 1, the retailer selects the retail price p_1 to maximize the total profit from both periods:

$$\max_{p_1} \left\{ (p_1 - w_1)(a_1 - bp_1) + \frac{1}{16b} \int_0^1 \left[a_2 - b(c_1 - \gamma x(a_1 - bp_1))\right]^2 f(x)dx \right\} \quad (4.13)$$

The first order condition of (4.13) with regard to p_1 gives us the best repones retail price $p_1(w_1)$ as a function of w_1:

$$p_1(w_1) = \frac{8(a_1 + bw_1) - (a_2 - bc_1)\gamma\mu + a_1 b^2\gamma^2(\mu^2 + \sigma^2)}{b\left[16 - b^2\gamma^2(\mu^2 + \sigma^2)\right]} \quad (4.14)$$

The manufacturer sets the wholesale price in the first period to maximize the total profit from the two periods:

$$\max_{w_1} \left\{ (w_1 - c_1)(a_1 - bp_1(w_1)) + \frac{1}{8b} \int_0^1 \left[a_2 - b(c_1 - \gamma x(a_1 - bp_1(w_1)))\right]^2 f(x)dx \right\} \quad (4.15)$$

Solve (4.15) for the equilibrium wholesale prices w_1^*. We obtain the equilibrium retail price p_1^* by substituting w_1^* into (4.20). Other equilibrium results are obtained using the following relationship: $q_1^* = a - bp_1^*$, $c_2^* = c_1 - \gamma\mu q_1^*$, $w_2^* = \frac{a+bc_2^*}{2b}$, $p_2^* = \frac{a_2+bw_2^*}{2b}$, and $q_2^* = a_2 - bp_2^*$. ■

Proof of Proposition 4.4.3

(i)-(ii) We can show the following relationships hold:

$$p_1^{D*} - p_1^{C*} = \frac{32(a_1 - bc_1) + b\gamma\mu\left[20 - b^2\gamma^2(\mu^2 + \sigma^2)\right](a_2 - bc_1)}{4b\left[8 - b^2\gamma^2(\mu^2 + \sigma^2)\right]\left[4 - b^2\gamma^2(\mu^2 + \sigma^2)\right]} > 0$$

$$q_1^{D*} - q_1^{C*} = b(p_1^{C*} - p_1^{D*}) < 0$$

$$E[p_2^{D*}] - E[p_2^{C*}] = \frac{32(a_1 - bc_1) + b\gamma\mu\left[20 - b^2\gamma^2(\mu^2 + \sigma^2)\right](a_2 - bc_1)}{4b\left[8 - b^2\gamma^2(\mu^2 + \sigma^2)\right]\left[4 - b^2\gamma^2(\mu^2 + \sigma^2)\right]} > 0$$

$$E[q_2^{D*}] - E[q_2^{C*}] = b(E[p_2^{C*}] - E[p_2^{D*}]) < 0$$

$$E[c_2^{D*}] - E[c_2^{C*}] = \gamma\mu(q_1^{C*} - q_1^{D*}) > 0$$

and

$$\pi_c^{D*} - \pi_c^{C*} = \left\{ \begin{array}{c} -\frac{(a_2-bc_1)^2(1024-64b^2\gamma^2\mu^2-256b^2\gamma^2\sigma^2+b^4\gamma^4\mu^4+17b^4\gamma^4\mu^2\sigma^2+16b^4\gamma^4\sigma^4)}{256b[8-b^2\gamma^2(\mu^2+\sigma^2)]^2} \\ -\frac{64b^2\gamma^2(a_1-bc_1)^2(\mu^2+\sigma^2)+16b\gamma\mu(a_1-bc_1)(a_2-bc_1)(32-b^2\gamma^2(\mu^2+\sigma^2))}{256b[8-b^2\gamma^2(\mu^2+\sigma^2)]^2} \end{array} \right\} < 0.$$

■

Proof of Theorem 4.4.4.

(i) The first order condition of the retailer's profit in Period 2 gives the best response $p_2(w_2) = \frac{a_2\phi_2+bw_2}{2b\phi_2}$. If the manufacturer sets $w_2 = \phi_2 c_2(x)$, then $p_2 = \frac{a_2+bc_2(x)}{2b} = \frac{a_2+b(c_1-\gamma x(a_1-bp_1))}{2b}$. The retailer's total profit from both periods is given by:

$$\pi_r = (\phi_1 p_1 - w_1)(a_1 - bp_1) + \phi_2 E\left[\pi_2^*(x)\right] \tag{4.16}$$

where $E[\pi_2^*(x)]$ is the second period profit in the centralized channel and is given by (4.12). Take the first order condition of (4.16) and solve for the best response $p_1(w_1)$:

$$p_1(w_1) = \frac{2(a_1\phi_1 + bw_1) - \phi_2 b\gamma\left(\mu(a_2 - bc_1) + a_1 b\gamma(\mu^2+\sigma^2)\right)}{b\left[4\phi_1 - \phi_2 b^2\gamma^2(\mu^2+\sigma^2)\right]}. \tag{4.17}$$

Equate (4.17) to p_1^* in the centralized case, and solve for w_1 to get the coordinating wholesale price w_1^* in Theorem 4.4.4. Under such a coordinating wholesale price contract, the first period retail price is equal to that in the centralized channel and the order quantity is equal to that in the centralized channel. The channel coordination is achieved in Period 1. In Period 2, if the manufacturer sets $w_2^* = \phi_2 c_2^*(x) = \phi_2(c_1 - \gamma x q_1^*)$, where $q_1^* = a_1 - bp_1^*$ and is given by Proposition 1(i), then the coordination is achieved in Period 2 as the second period retail price, order quantity, wholesale price are equal to those in the centralized channel, respectively.

(ii) It is straightforward by substituting $\phi_1 = \phi_2 = \phi$ into w_1^* to get $w_1^* = \phi c_1$. ■

Proof of Proposition 4.4.5.

We have $\frac{\partial w_2^*(x)}{\partial \phi_1} = 0$ and $\frac{\partial w_2^*(x)}{\partial \phi_2} > 0$. The following holds:

$$\frac{\partial w_1^*}{\partial \phi_2} = \frac{\gamma\left(2\mu(a_2 - bc_1) + b\gamma(a_1 - bc_1)(\mu^2+\sigma^2)\right)}{4 - b^2\gamma^2(\mu^2+\sigma^2)} > 0$$

$$\frac{\partial \pi_r^*}{\partial \phi_1} = -\frac{\partial \pi_m^*}{\partial \phi_1} = \frac{\left(2(a_1 - bc_1) + b\gamma\mu(a_2 - bc_1)\right)^2}{b\left(4 - b^2\gamma^2(\mu^2+\sigma^2)\right)^2} > 0$$

and

$$\frac{\partial \pi_r^*}{\partial \phi_2} = -\frac{\partial \pi_m^*}{\partial \phi_2} = \left\{ \begin{array}{c} \frac{\left[16-8b^2\gamma^2(\mu^2+\sigma^2)+b^4\gamma^4\sigma^2(\mu^2+\sigma^2)\right](a_2-bc_1)^2}{4b(4-b^2\gamma^2(\mu^2+\sigma^2))^2} \\ -\frac{4b^2\gamma^2(\mu^2+\sigma^2)(a_1-bc_1)^2+4b^3\gamma^3\mu(\mu^2+\sigma^2)(a_1-bc_1)(a_2-bc_1)}{4b(4-b^2\gamma^2(\mu^2+\sigma^2))^2} \end{array} \right\}$$

If $a_2 \geqslant \Phi(c_1) \equiv bc_1 + \frac{2bc_1(1-b\gamma)\sqrt{\mu^2+\sigma^2}}{4-b^2\gamma^2(\mu^2+\sigma^2)-b^2\gamma^2\mu\sqrt{\mu^2+\sigma^2}}$, then $\frac{\partial \pi_r^*}{\partial \phi_2} = -\frac{\partial \pi_m^*}{\partial \phi_2} > 0$; if $a_2 < \Phi(c_1)$, then if $a_1 \leqslant \Psi(a_2) \equiv bc_1 + \frac{b\gamma(a_2-bc_1)\left[4-b^2\gamma^2(\mu^2+\sigma^2)-b^2\gamma^2\mu\sqrt{\mu^2+\sigma^2}\right]}{2b^2\gamma^2\sqrt{\mu^2+\sigma^2}}$, then $\frac{\partial \pi_r^*}{\partial \phi_2} = -\frac{\partial \pi_m^*}{\partial \phi_2} > 0$; otherwise, $\frac{\partial \pi_r^*}{\partial \phi_2} = -\frac{\partial \pi_m^*}{\partial \phi_2} < 0$. ∎

Proof of Lemma 4.5.1.

We fist assume that $I_2 > 0$. Later we will provide the condition under which this actually occurs. The retailer's profit in Period 2:

$$\pi_2(p_2, q_2|x) = p_2\left(a_2 - b_2p_2\right) - c_2\left(x\right)\left(a_2 - b_2p_2 - I_2\right) - hI_2$$

The first order condition of the above equation gives $p_2\left(x\right) = \frac{a_2+bc_2(x)}{2b}$. Substitute $p_2(x)$ and take the expected second-period profit $E\left[\pi_2^*(p_1,q_1)\right] = \int_0^1 \pi_2(p_2,q_2|x)f(x)dx$:

$$\begin{aligned} E\left[\pi_2^*(p_1,q_1)\right] &= \int_0^1 \left[p_2(x)\left(a_2 - bp_2(x)\right) - c_2\left(x\right)\left(a_2 - bp_2(x) - I_2\right)f(x)\right]dx - hI_2 \\ &= \frac{1}{4b}\int_0^1 \left(a_2 - bc_2(x)\right)^2 f(x)dx + \int_0^1 c_2(x)I_2f(x)dx - hI_2 \\ &= \frac{1}{4b}\int_0^1 \left(a_2 - b\left(c_1 - \gamma x q_1\right)\right)^2 f(x)dx + \int_0^1 \left(c_1 - \gamma x q_1\right)\left[q_1 - \left(a_1 - bp_1\right)\right]f(x)dx \\ &\quad -h\left[q_1 - \left(a_1 - bp_1\right)\right] \\ &= \frac{1}{4b}\left\{\left(a_2 - bc_1\right)^2 + 2\gamma\mu b_2\left[a_2 - bc_1\right]q_1 + \left(b\gamma q_1\right)^2 E\left[x^2\right]\right\} \\ &\quad +\left[q_1 - \left(a_1 - bp_1\right)\right]\left(c_1 - \gamma\mu q_1 - h\right) \end{aligned} \tag{4.18}$$

The first-period profit is given by

$$\pi = p_1\left(a_1 - bp_1\right) - c_1q_1 + E\left[\pi_2^*(p_1,q_1)\right],$$

where $E\left[\pi_2^*(p_1,q_1)\right]$ is given by (4.18). Take the first order condition of π with regard to p_1

and q_1, respectively:

$$\frac{\partial \pi}{\partial p_1} = a_1 - 2bp_1 + b_1\left(c_1 - \gamma\mu q_1 - h\right) = 0$$
$$\frac{\partial \pi}{\partial q_1} = -h + \frac{1}{2}bq_1\gamma^2 E\left[x^2\right] + \gamma\mu\left[a_1 + \frac{1}{2}\left(a_2 - bc_1\right) - bp_1 - 2q_1\right] = 0$$

Solving the above two equations simultaneously to get the equilibrium retail price and quantity in Period 1 as shown in Lemma 4.5.1. The optimal inventory I_2^* is given by

$$\begin{aligned} I_2^* &= q_1^* - (a_1 - b_1 p_1^*) \\ &= \frac{\gamma\left[\mu(2 - b\gamma\mu)(a_2 - a_1) + b\gamma\sigma^2(a_1 - bc_1)\right] - h\left[4 - b^2\gamma^2(\mu^2 + \sigma^2)\right]}{2\gamma(4\mu - 2b\gamma\mu^2 - b\gamma\sigma^2)}. \end{aligned}$$

Under our assumptions, we have $(4\mu - 2b\gamma\mu^2 - b\gamma\sigma^2) > 0$ and $4 - b^2\gamma^2(\mu^2 + \sigma^2) > 0$. Let

$$S_{cn} = -h\left[4 - b^2\gamma^2(\mu^2 + \sigma^2)\right] + \gamma\left[\mu(2 - b\gamma\mu)(a_2 - a_1) + b\gamma\sigma^2(a_1 - bc_1)\right]$$

If $S_{cn} > 0$, we have $I_2^* > 0$; otherwise $I_2^* = 0$. ■

Proof of Lemma 4.5.2.

We consider two cases: $I_2^* > 0$ and $I_2^* = 0$, where the latter case is solved in Section 4.2. We first assume that the inventory is carried and proceed to solve the problem. Then we provide conditions for this to hold. In the second period, the best response retail price is $p = (a_2 + b_2 w_2)/2b$. The manufacturer solves the second period problem to get $w_2(x) = (a_2 + bc_2(x) - 2I_2)/2b$, where $c_2(x) = c_1 - q_1\gamma x$. We obtain the second period profits for retailer and the manufacturer as:

$$\pi_{r2}^*(x) = \frac{1}{16b}\left[(a_2 - bc_2(x))^2 + 4I_2(3a_2 + b(c_2(x) - 4h)) - 12I_2^2\right]$$
$$\pi_{m2}^*(x) = \frac{1}{8b}(a_2 - bc_2(x) - 2I_2)^2$$

where $c_2(x) = c_1 - q_1\gamma x$ and $I_2 = q_1 - (a_1 - bp_1)$. In the first period, the retailer solves the following problem:

$$\pi_r(p_1, q_1) = \pi_{r1}(p_1, q_1) + \frac{1}{16b}\int_0^1 \left[(a_2 - bc_2(x))^2 + 4I_2(3a_2 + bc_2(x)) - 12I_2^2\right] f(x)dx$$

where $\pi_{r1}(p_1, q_1) = p_1(a_1 - bp_1) - w_1 q_1$. Take the first order condition of $\pi_r(p_1, q_1)$ with regard to p_1 and q_1. Set $\partial\pi_r(p_1, q_1)/\partial p_1 = 0$ and $\partial\pi_r(p_1, q_1)/\partial q_1 = 0$ and solve the two equations simultaneously to get the best responses $p_1(w_1)$ and $q_1(w_1)$ as follows:

$$q_1(w_1) = \frac{8(3a_1 + 3a_2 + b(c_1 - 4h - 7w_1) + 4b(a_1 + a_2 + b(-2c_1 + h))\gamma\mu}{48 - b\gamma\left(8\mu(-2 + b\gamma\mu) + 7b\gamma\sigma^2\right)} \tag{4.19}$$

$$\begin{aligned} p_1(w_1) = \; & \frac{4b^2\gamma^2\mu^2(3a_1 + a_2 - bh) + b^2\gamma^2\sigma^2(10a_1 + 3a_2 + b(c_1 - 4h))}{2b\left(-48 + b\gamma\left(8\mu(-2 + b\gamma\mu) + 7b\gamma\sigma^2\right)\right)} \\ & - \frac{48(a_1 + bw_1) + 8b(2a_1 + b(c_1 - h + w_1))\gamma\mu}{2b\left(-48 + b\gamma\left(8\mu(-2 + b\gamma\mu) + 7b\gamma\sigma^2\right)\right)} \end{aligned} \tag{4.20}$$

Substitute (4.19) and (4.20) into $c_2(x) = c_1 - q_1\gamma x$ and $I_2 = q_1 - (a_1 - bp_1)$. The manufacturer's first-period problem is:

$$\pi_m^* = \max_{w_1} \left\{ \pi_m(w_1) = (w_1 - c_1)q_1(w_1) + \frac{1}{8b}\int_0^1 \left[a_2 - bc_2(x) - 2I_2\right]^2 f(x)dx \right\} \tag{4.21}$$

The first order condition of $\pi_m(w_1)$ in (4.21) with regard to w_1 gives the equilibrium w_1^*:

$$w_1^* = \left\{ \begin{array}{l} \frac{32(9a_1+9a_2+16bc_1)+16b(17a_1+17a_2+26bc_1)\gamma\mu}{8b(136+120b\gamma\mu-60b^2\gamma^2\mu^2-49b^2\gamma^2\sigma^2)} \\ -\frac{2b^2\gamma^2\left(80a_1\mu^2+80a_2\mu^2+80bc_1\mu^2+71a_1\sigma^2+71a_2\sigma^2+54bc_1\sigma^2\right)}{8b(136+120b\gamma\mu-60b^2\gamma^2\mu^2-49b^2\gamma^2\sigma^2)} \\ -\frac{b^3(a_1+a_2-2bc_1)\gamma^3\mu\left(8\mu^2+5\sigma^2\right)}{8b(136+120b\gamma\mu-60b^2\gamma^2\mu^2-49b^2\gamma^2\sigma^2)} \\ -\frac{bh\left(128+464b\gamma\mu-208b^2\gamma^2\mu^2+8b^3\gamma^3\mu^3-152b^2\gamma^2\sigma^2+5b^3\gamma^3\mu\sigma^2\right)}{8b(136+120b\gamma\mu-60b^2\gamma^2\mu^2-49b^2\gamma^2\sigma^2)} \end{array} \right\} \tag{4.22}$$

Substitute (4.22) into (4.19) and (4.20) to get the equilibrium retail price as

$$p_1^* = \left\{ \begin{array}{l} \frac{4(43a_1+9a_2+16bc_1)+4b(37a_1+7a_2+16bc_1)\gamma\mu}{2b(136+120b\gamma\mu-60b^2\gamma^2\mu^2-49b^2\gamma^2\sigma^2)} \\ +\frac{-b^2\gamma^2\left(89a_1\mu^2+29a_2\mu^2+2bc_1\mu^2+70a_1\sigma^2+21a_2\sigma^2+7bc_1\sigma^2\right)}{2b(136+120b\gamma\mu-60b^2\gamma^2\mu^2-49b^2\gamma^2\sigma^2)} \\ +\frac{bh\left(-16-78b\gamma\mu+31b^2\gamma^2\mu^2+28b^2\gamma^2\sigma^2\right)}{2b(136+120b\gamma\mu-60b^2\gamma^2\mu^2-49b^2\gamma^2\sigma^2)} \end{array} \right\} \tag{4.23}$$

and the first-period order quantity q_1^* as

$$q_1^* = \frac{(a_1 + a_2 - 2bc_1)(26 + 23b\gamma\mu) + bh(-72 + 23b\gamma\mu)}{136 + 120b\gamma\mu - 60b^2\gamma^2\mu^2 - 49b^2\gamma^2\sigma^2} \tag{4.24}$$

Substitute (4.23) and (4.24) into $E[c_2(x)] = c_1 - q_1^*\gamma\mu$ to get

$$\begin{aligned} E[c_2^*] &= c_1 - q_1^*\gamma\mu \\ &= \left\{ \begin{array}{l} \frac{136c_1 - 2\gamma\mu(13a_1 + 13a_2 - 86bc_1) + bh\gamma\mu(72 - 23b\gamma\mu)}{136 + 120b\gamma\mu - 60b^2\gamma^2\mu^2 - 49b^2\gamma^2\sigma^2} \\ -\frac{b\gamma^2\left(23a_1\mu^2 + 23a_2\mu^2 + 14bc_1\mu^2 + 49bc_1\sigma^2\right)}{136 + 120b\gamma\mu - 60b^2\gamma^2\mu^2 - 49b^2\gamma^2\sigma^2} \end{array} \right\} \end{aligned}$$

and into $I_2^* = q_1^* - (a_1 - bp_1^*)$ to get I_2^* in(), and into $E[w_2^*] = (a_2 + bE[c_2(x) - 2I_2^*])/2b$ to get

$$\begin{aligned} E[w_2^*] &= \frac{a_2 + bE[c_2(x) - 2I_2^*}{2b} \\ &= \left\{ \begin{array}{l} \frac{8(3a_1 + 3a_2 + 11bc_1) + 10b(a_1 + a_2 + 10bc_1)\gamma\mu}{b(136 + 120b\gamma\mu - 60b^2\gamma^2\mu^2 - 49b^2\gamma^2\sigma^2)} \\ + \frac{-b^2\gamma^2\left(27a_1\mu^2 + 27a_2\mu^2 + 6bc_1\mu^2 + 14a_1\sigma^2 + 14a_2\sigma^2 + 21bc_1\sigma^2\right)}{b(136 + 120b\gamma\mu - 60b^2\gamma^2\mu^2 - 49b^2\gamma^2\sigma^2)} \\ + \frac{bh\left(80 + 52b\gamma\mu - 27b^2\gamma^2\mu^2 - 14b^2\gamma^2\sigma^2\right)}{b(136 + 120b\gamma\mu - 60b^2\gamma^2\mu^2 - 49b^2\gamma^2\sigma^2)} \end{array} \right\} \end{aligned}$$

$$\begin{aligned} E[p_2^*] &= \frac{8(3a_1 + 20a_2 + 11bc_1) + 10b(a_1 + 13a_2 + 10bc_1)\gamma\mu}{2b\left(136 + 120b\gamma\mu - 60b^2\gamma^2\mu^2 - 49b^2\gamma^2\sigma^2\right)} \\ &\quad + \frac{-b^2\gamma^2\left(27a_1\mu^2 + 87a_2\mu^2 + 6bc_1\mu^2 + 14a_1\sigma^2 + 63a_2\sigma^2 + 21bc_1\sigma^2\right)}{2b\left(136 + 120b\gamma\mu - 60b^2\gamma^2\mu^2 - 49b^2\gamma^2\sigma^2\right)} \\ &\quad + \frac{bh\left(80 + 52b\gamma\mu - 27b^2\gamma^2\mu^2 - 14b^2\gamma^2\sigma^2\right)}{2b\left(136 + 120b\gamma\mu - 60b^2\gamma^2\mu^2 - 49b^2\gamma^2\sigma^2\right)} \end{aligned}$$

$$\begin{aligned} E[q_2^*] &= \frac{(a_1 + a_2 - 2bc_1)\left(12 + 18b\gamma\mu - 2b^2\gamma^2\mu^2 - 7b^2\gamma^2\sigma^2\right)}{136 + 120b\gamma\mu - 60b^2\gamma^2\mu^2 - 49b^2\gamma^2\sigma^2} \\ &\quad + \frac{bh\left(40 - 10b\gamma\mu - 2b^2\gamma^2\mu^2 - 7b^2\gamma^2\sigma^2\right)}{136 + 120b\gamma\mu - 60b^2\gamma^2\mu^2 - 49b^2\gamma^2\sigma^2} \end{aligned}$$

$$\begin{aligned} E[c_2^*] &= \frac{136c_1 - 2\gamma\mu(13a_1 + 13a_2 - 86bc_1) + bh\gamma\mu(72 - 23b\gamma\mu)}{136 + 120b\gamma\mu - 60b^2\gamma^2\mu^2 - 49b^2\gamma^2\sigma^2} \\ &\quad - \frac{b\gamma^2\left(23a_1\mu^2 + 23a_2\mu^2 + 14bc_1\mu^2 + 49bc_1\sigma^2\right)}{136 + 120b\gamma\mu - 60b^2\gamma^2\mu^2 - 49b^2\gamma^2\sigma^2} \end{aligned}$$

Use $I_2^* = q_1^* - (a_1 - bp_1^*)$ to get I_2^* expressed in Lemma 4.5.2. Note that $I_2^* > 0$ when h satisfies the condition in Lemma 4.5.2. Otherwise, $I_2^* = 0$ and the solutions are provided in section 4.4. ∎

Proof of Theorem 4.5.3.

Suppose that the manufacturer proposes $w_2^*(x) = \phi_2 c_2(x)$. The retailer's optimal profit in Period 2 is given by:

$$E[\pi_{r2}^*] = \int_0^1 (\phi_2 p_2^*(x)(a_2 - b_2 p_2^*(x)) - w_2^*(x)(a_2 - b_2 p_2^*(x) - I_2)) f(x) dx \tag{4.25}$$

where $p_2^*(x) = (a_2 + b_2 c_2(x))/2b_2$ and $I_2 = I_1 + q_1 - (a_1 - b_1 p_1)$.

In Period 1, the retailer's problem:

$$\max_{p_1 > 0, q_1 > 0} \pi_r (p_1, q_1) = \phi_1 p_1 (a_1 - b_1 p_1) - w_1 q_1 + E[\pi_{r2}^*]$$

where $E[\pi_{r2}]$ is given by (4.25). Solving the problem gives the first period retail price

$$p_1^* = \left\{ \begin{array}{c} \frac{(4a_1\phi_1 + 2bw_1 + 2bc_1\phi_2)\mu + b\phi_2(bc_1 - 2a_1 - a_2)\gamma\mu^2}{2b(4\phi_1\mu - \phi_1 b\gamma(\mu^2+\sigma^2) - \phi_2 b\gamma\mu^2)} \\ \frac{-b(a_1\phi_1 + bc_1\phi_2)\gamma(\mu^2+\sigma^2) - bh(2\mu - b\gamma(\mu^2+\sigma^2))}{2b(4\phi_1\mu - \phi_1 b\gamma(\mu^2+\sigma^2) - \phi_2 b\gamma\mu^2)} \end{array} \right\} \tag{4.26}$$

and the order quantity

$$q_1^* = \frac{((a_1 + a_2 - bc_1)\phi_1 - bc_1\phi_2)\phi_2\gamma\mu - 2\phi_1(w_1 - c_1\phi_2) - h(2\phi_1 - b\phi_2\gamma\mu)}{\phi_2\gamma(4\phi_1\mu - b\phi_1\gamma(\mu^2+\sigma^2) - b\phi_2\gamma\mu^2)}. \tag{4.27}$$

In order to coordinate the supply chain, the retail price and order quantity in the first period should be the same as that in the centralized channel. Equating (4.26) to p_1^* in the centralized case, and (4.27) to q_1^* in the centralized case yields:

$$w_1^* = c_1\phi_2 - h(1 - \phi_2)$$

and

$$\phi_1 = \frac{h(-4\mu + b\gamma(2\mu^2 + \sigma^2)) + (A + (2 - b\mu\gamma)\mu h)\phi_2}{A - h(2\mu - b\gamma(\mu^2 + \sigma^2))}.$$

where $A = -\gamma\mu^2(a_1 + a_2) + c_1(4\mu - b\gamma\sigma^2)$. ∎

Proof of Proposition 4.6.1.

If $\gamma = 0$ and $a_1 = a_2 = a$, from Table 4.3 we know that only when $0 \leqslant h < (a - bc_1)/4$ will the retailer carry over inventory. Therefore, when $h \geqslant (a-bc_1)/4$, $\pi_m^{I*} = \pi_m^{N*}$, $\pi_r^{I*} = \pi_r^{N*}$ and $\pi_c^{I*} = \pi_c^{N*}$. When $0 \leqslant h < (a-bc_1)/4$, it can be easily verified that

$$\pi_m^{I*} - \pi_m^{N*} = \frac{(-a_2 + bc_1 + 4bh)^2}{68b} > 0,$$

$$\pi_r^{I*} - \pi_r^{N*} = \frac{(-a_2 + bc_1 + 4bh)(-21a_2 + 21bc_1 + 152bh)}{2312b} \begin{cases} > 0, & \text{if } 0 \leqslant h < \frac{21(a-bc_1)}{152b}, \\ \leqslant 0, & \text{if } \frac{21(a-bc_1)}{152b} \leqslant h < \frac{a-bc_1}{4b}, \end{cases}$$

and

$$\pi_c^{I*} - \pi_c^{N*} = \frac{(-a_2 + bc_1 + 4bh)(-55a_2 + 55bc_1 + 288bh)}{2312b} \begin{cases} > 0, & \text{if } 0 \leqslant h < \dfrac{55(a-bc_1)}{288b}, \\ \leqslant 0, & \text{if } \dfrac{55(a-bc_1)}{288b} \leqslant h < \dfrac{a-bc_1}{4b}. \end{cases}$$

This completes the proof. ■

Proof of Proposition 4.6.2.

If $h = 0$ and $a_1 = a_2 = a$, then

$$\pi_m^{I*} - \pi_m^{N*} = \frac{(a-bc_1)^2(256 + 160b\gamma - 2b^2\gamma^2 - 14b^3\gamma^3 - b^4\gamma^4)}{64b(8 - b^2\gamma^2)(34 + 30b\gamma - 15b^2\gamma^2)} > 0;$$

Let:

$$\begin{aligned} A_1 = G(b\gamma) = \; & 172032 - 183296b\gamma - 462144b^2\gamma^2 + 35648b^3\gamma^3 \\ & +124772b^4\gamma^4 - 1544b^5\gamma^5 - 9736b^6\gamma^6 + 12b^7\gamma^7 + 121b^8\gamma^8 \\ A_2 = F(b\gamma) = \; & 450560 + 236544b\gamma - 468416b^2\gamma^2 - 110784b^3\gamma^3 + 107636b^4\gamma^4 \\ & +15960b^5\gamma^5 - 7560b^6\gamma^6 - 708b^7\gamma^7 + 61b^8\gamma^8 \end{aligned}$$

We find that $G_0 \approx 0.4579$. For $0 < b\gamma < 1$, we have $A_2 > 0$. So we have:

$$\begin{aligned} \pi_r^{I*} - \pi_r^{N*} &= \frac{A_1(a-bc_1)^2}{256b(8-b^2\gamma^2)^2(34+30b\gamma-15b^2\gamma^2)^2} = \begin{cases} > 0, & \text{if } 0 \leqslant b\gamma < G_0; \\ < 0, & \text{if } G_0 < b\gamma < 1; \end{cases} \\ \pi_c^{I*} - \pi_c^{N*} &= \frac{A_2(a-bc_1)^2}{256b(8-b^2\gamma^2)^2(34+30b\gamma-15b^2\gamma^2)^2} > 0. \end{aligned}$$

■

Proof of Proposition 4.6.3.

If $h = 0$ and $\gamma = 0$, then from Table 4.5 we know that only when $a_2 > (6a_1 + 5bc_1)/11$ will the retailer carry over inventory. Therefore, when $a_2 \leqslant (6a_1 + 5bc_1)/11$, $\pi_m^{I*} = \pi_m^{N*}$, $\pi_r^{I*} = \pi_r^{N*}$ and $\pi_c^{I*} = \pi_c^{N*}$. When $a_2 > (6a_1 + 5bc_1)/11$, it can be easily verified that

$$\pi_r^{I*} - \pi_r^{N*} = \frac{3\left(74(a_1 - a_2)^2 + 7(a_1 - bc_1)(a_2 - bc_1)\right)}{2312b} > 0,$$

$$\pi_c^{I*} - \pi_c^{N*} = \frac{\left(86(a_1 - a_2)^2 + 55(a_1 - bc_1)(a_2 - bc_1)\right)}{2312b} > 0,$$

and

$$\begin{aligned}\pi_m^{I*} - \pi_m^{N*} &= \frac{-4a_1^2 + 9a_1a_2 - 4a_2^2 - a_1bc_1 - a_2bc_1 + b^2c_1^2}{68b} \\ &\begin{cases} < 0, & \text{if } a_2 > \frac{9a_1 - bc_1 + \sqrt{17}(a_1 - bc_1)}{8}, \\ \geqslant 0, & \text{if } \frac{9a_1 - bc_1 - \sqrt{17}(a_1 - bc_1)}{8} \leqslant a_2 \leqslant \frac{9a_1 - bc_1 + \sqrt{17}(a_1 - bc_1)}{8}, \\ < 0, & \text{if } \frac{6a_1 + 5bc_1}{11} < a_2 < \frac{9a_1 - bc_1 - \sqrt{17}(a_1 - bc_1)}{8}. \end{cases}\end{aligned}$$

This completes the proof. ■

REFERENCES

Agrawal, N., S. Nahmias. 1997. Rationalization of the Supplier Base in the Presence of Yield Uncertainty. *Production and Operations Management*, **6**(3), 291–308.

Alvarez, F., H. Amman. 1999. Learning-By-Doing under Uncertainty. *Computational Economics* **14** 255-262.

Anand, K., R. Anupindi, Y. Bassok. 2008. Strategic inventories in vertical contracts. *Management Science* **54**(10) 1792-1804.

Anupindi, R., R. Akella. 1993. Diversification Under Supply Uncertainty. *Management Science*, **39**(8), 944–963.

Arrow, K.J. 1962. The economic implications of learning by doing. *Review of Economic Studies* **29**(3) 155-173.

Babich, V., A. N. Burnetas, P. H. Ritchken. 2007. Competition and Diversification Effects in Supply Chains with Supplier Default Risk. *Manufacturing & Service Operations Management*, **9**(2), 123–146.

Baloff, N. 1971. Extension of the learning curve- Some empirical results. *Operational Research Quarterly* **22**(4) 329-340.

Bish, E. K., Q. Wang. 2004. Optimal Investment Strategies for Flexible Resources, Considering Pricing and Correlated Demands. *Operations Research*, **52**(6), 954–964.

Boyacı, T., Ö. Özer. 2010. Information Acquisition for Capacity Planning via Pricing and Advance Selling: When to Stop and Act? *Operations Research*, **58**(5), 1328–1349.

Burke, G. J., J. E. Carrillo, A. J. Vakharia. 2009. Sourcing Decisions with Stochastic Supplier Reliability and Stochastic Demand. *Production and Operations Management*, **18**(4), 475–484.

Cachon, G., M. A. Lariviere. 2005. Supply chain coordination with revenue-sharing contracts: Strengths and limitations. *Management Science* **51**(1) 30-44.

Chand, S., S. P. Sethi. 1990. A dynamic lot sizing model with learning in setups. *Operations Research*(38) 644-655.

Chod, J., N. Rudi. 2005. Resource Flexibility with Responsive Pricing. *Operations Research*, **53**(3), 532–548.

Ciarallo, F. W., R. Akella, T. E. Morton. 1994. A Periodic Review Production Planning Model with Uncertain Capacity and Uncertain Demand-Optimality of Extended Myopic Policies. *Management Science*, **40**(3), 320–332.

Clark, D., Y. Takahashi. 2011. Quake Disrupts Key Supply Chains. *Wall Street Journal* (March 12).

Dada, M., N. Petruzzi, L. B. Schwarz. 2007. A Newsvendor's Procurement Problem When Suppliers are Unreliable. *Manufacturing & Service Operations Management*, **9**(1), 9–32.

Dasgupta, P., J. Stiglitz. 1988. Learning-by-doing, market structure and industrial and trade policies. *Oxford Economic Papers* **40** 246-268.

Dawson, C.. 2011. Quake Still Rattles Suppliers. *Wall Street Journal* (September 29).

Dawson, C., Y. Takahashi. 2011. Toyota Signals A Bleak Year. *Wall Street Journal* (April 23).

Einhorn, B., T. Culpan, A. Ohnsman. 2011. Now, A Weak Link in the Global Supply Chain. *Bloomberg Businessweek* (March 17).

Erhun, F., P. Keskinocak, S. Tayur. 2008. Dynamic procurement in a capacitated supply chain facing uncertain demand. *IIE Transaction* **40** 733-748.

Federgruen, A., N. Yang. 2009. Optimal Supply Diversification Under General Supply Risks. *Operations Research*, **57**(6), 1451–1468.

Feng, Q. 2010. Integrating Dynamic Pricing and Replenishment Decisions Under Supply Capacity Uncertainty. *Management Science*, **56**(12), 2154–2172.

Feng, Q., R. Shi. 2012. Sourcing from Multiple Suppliers for Price-Dependent Demands. *Production and Operations Management*, **21**(3), 547–563.

Fudenberg, D., J. Tirole. 1983. Learning-by-doing and market performance. *The RAND Journal of Economics.* **14**(2) 522-530.

Gerchak, Y., R. K. Cho, and S. Ray. 2006. Coordination of quantity and shelf-retention timing in the video movie rental industry. *IIE Trans*action. **38** 525-536.

Gerchak, Y., M. Parlar. 1990. Yield Variability, Cost Tradeoffs and Diversification in the EOQ Model. *Naval Research Logistics*, **37**(5), 341–354.

Gerchak, Y., Y. Wang. 2004. Revenue-sharing vs. wholesale-price contracts in assembly systems with random demand. *Production and Operations Management* **13**(1) 23-33.

Giannoccaro, I., P. Pontrandolfo. 2004. Supply chain coordination by revenue sharing contracts. *Int. J. Production Economics* **89** 131-139

Grosfeld-Nir, A., Y. Gerchak. 2004. Multiple Lotsizing in Production to Order with Random Yields: Review of Recent Advances. *Annals of Operations Research*, **126**, 43–69.

Gutierrez, G. J., X. He. 2011. Life-cycle Channel Coordination Issues in Launching an Innovative Durable Product. *Production and Operations Management*, **20**(2) 268-279.

Hatch, N. W., D. C. Mowery. 1998. Process innovation and learning by doing in semiconductor manufacturing. *Management Sci.* **44**(11) 1461-1477.

He, X., T. Li, S. P. Sethi. 2012. Strategic Inventories and Dynamic Coordination with Production Cost Learning *Working Paper*. The University of Texas at Dallas, Dallas, TX.

Hiller, R. S., J. F. Shapiro. 1986. Optimal capacity expansion planning when there are learning effects. *Management Sci.* **32**(9) 1153-1163.

Johnson, N. L., S. Kotz. 1975. A Vector Multivariate Hazard Rate. *Journal of Multivariate Analysis*, **5**(1), 53–66.

Jørgensen, S., P. M. Kort, G. Zaccour. 1999. Production, inventory, and pricing under cost and demand learning effects. *European Journal of Operational Research*. **117** 382-395.

Karia, S. R., J. V. Deshpande. 1999. Bounds for the Hazard Gradients in the Competing Risks Set Up. *Journal of Statistical Planning and Inference*, **75**(2), 363–377.

Karwan, P. B., J. B. Mazzola, R. C. Morey. 1988. Production lot sizing under setup and worker learning. *Naval Research Logistics* **35** 159-175.

Kazaz, B. 2004. Production Planning under Yield and Demand Uncertainty with Yield-Dependent Cost and Price. *Manufacturing & Service Operations Management*, **6**(3), 209–224.

Latour, A. 2001. Trial by Fire: A Blaze in Albuquerque Sets Off Major Crisis for Cell-Phone Giants. *Wall Street Journal* (January 29).

Lee, H. L., V. Padmanabhan, T. A. Taylor, S. Whang. 2000. Price protection in the personal computer industry. *Management Sci.* **46**(4) 467-482.

Li, Q., S. Zheng. 2006. Joint Inventory Replenishment and Pricing Control for Systems with Uncertain Yield and Demand. *Operations Research*, **54**(4), 696–705.

Li, T., S. P. Sethi, J. Zhang. 2012a. Supply Diversification with Responsive Pricing. *Working Paper*. The University of Texas at Dallas, Dallas, TX.

Li, T., S. P. Sethi, J. Zhang. 2012b. How Does Pricing Power Affect A Firm's Sourcing Decisions from Unreliable Suppliers? *Working Paper*. The University of Texas at Dallas, Dallas, TX.

Linh, C. T., Y. Hong. 2009. Channel coordination through a revenue sharing contract in a two-period newsboy problem. *European Journal of Operational Research*. **198** 822-829.

Majd, S., R. S. Pindyck. 1989. The learning curve and optimal production under uncertainty. *Rand Journal of Economics* **20**(3) 331-343.

Marshall A. W. 1975. Some Comments on the Hazard Gradient. *Stochastic Processes and Their Applications*, **3**(3), 293–300.

Mazzola, J. B., K. F. McCardle. 1996. A Bayesian approach to managing learning-curve uncertainty. *Management Sci*. **42**(5) 680-692.

Mazzola, J. B., K. F. McCardle. 1997. The stochastic learning curve: Optimal production in the presence of learning curve uncertainty. *Operations Research*. **45**(3) 440-450.

Mills, E. S. 1959. Uncertainty and Price Theory. *Quarterly Journal of Economics*, **73**(1), 116–130.

Mills, E. S. 1962. *Price, Output and Inventory Policy*. John Wiley, New York.

Morse, W. J. 1972. Reporting production costs that follow the learning curve phenomenon. *The Accounting Review*. **47**(4) 761-773.

Özer, Ö., O. Uncu. 2012. Competing on Time: An Integrated Framework to Optimize Dynamic Time-to-Market and Production Decisions. *Production and Operations Management*. Forthcoming.

Oh, S., Ö. Özer. 2012. Mechanism Design for Capacity Planning under Dynamic Evolutions of Asymmetric Demand Forecasts. *Management Science*. Forthcoming.

Parlar, M., D. Wang. 1993. Diversification Under Yield Randomness in Two Simple Inventory Models. *European Journal of Operational Research*, **66**(1), 52–64.

Petruzzi, N., M. Dada. 1999. Pricing and the Newsvendor Problem: A Review with Extensions. *Operations Research*, **47**(2), 183–194.

Petruzzi, N. C., M. Dada. 2002. Dynamic pricing and inventory control with learning. *Naval Research Logistics* . **49** 303-325.

Porteus, E. L. 1990. Stochastic Inventory Theory. D. P. Heyman, M. J. Sobel, eds., *Stochastic Models*. North Holland, Amsterdam, The Netherlands, 605–652.

Ramasesh, R., J. K. Ord, J. C. Hayya, A. Pan. 1991. Sole Versus Dual Sourcing in Stochastic Leadtime (s, Q) Inventory Models. *Management Science*, **37**(4), 428–443.

Rotemberg, J. J., G. Saloner. 1989. Cyclical behavior of strategic inventories. *Quarterly Journal of Economics*. **104**(1) 73-97.

Saloner, G. 1986. The role of obsolescence and inventory costs in providing commitment. *International Journal of Industrial Organization* **4** 333-345.

Shaked, M., J. G. Shanthikumar. 2007. *Stochastic Orders*. Springer, New York.

Spence, A. M. 1981. The learning curve and competition. *Bell Journal of Economics* **12**(1) 49-70.

Swaminathan, J. M., J. G. Shanthikumar. 1999. Supplier Diversification: Effect of Discrete Demand. *Operations Research Letters*, **24**(5), 213–221.

Tang, C. S., R. Yin. 2007. Responsive Pricing Under Supply Uncertainty. *European Journal of Operational Research*, **182**(1), 239–255.

Taylor, T.A. 2001. Channel coordination under price protection, midlife returns, and end-of-life returns in dynamic markets. *Management Sci.* **47**(9) 1220-1234.

Tomlin, B. 2006. On the Value of Mitigation and Contingency Strategies for Managing Supply Chain Disruption Risks. *Management Science*, **52**(5), 639–657.

Tomlin, B. 2009. Impact of Supply Learning When Suppliers are Unreliable. *Manufacturing & Service Operations Management*, **11**(2), 192–209.

Tomlin, B. T., L. V. Snyder. 2007. On the Value of A Threat Advisory System for Managing Supply Chain Disruptions. *Working Paper*. University of North Carolina-Chapel Hill, Chapel Hill, NC.

Tomlin, B., Y. Wang. 2005. On the Value of Mix Flexibility and Dual Sourcing in Unreliable Newsvendor Networks. *Manufacturing & Service Operations Management*, **7**(1), 37–57.

Tomlin, B., Y. Wang. 2008. Pricing and Operational Recourse in Coproduction Systems. *Management Science*, **54**(3), 522–537.

Van Mieghem, J. A., M. Dada. 1999. Price versus Production Postponement: Capacity and Competition. *Management Science*, **45**(12), 1631–1649.

Wang, Y. 2008. Essays on Supply Risk in Global Operations. PhD Dissertation, The University of North Carolina at Chapel Hill.

Wang, Y., W. Gilland, B. Tomlin. 2010. Mitigating Supply Risk: Dual Sourcing or Process Improvement. *Manufacturing & Service Operations Management*, **12**(3), 489–510.

Whitin, T. M. 1955. Inventory Control and Price Theory. *Management Science*, **2**(1), 61–68.

Wolfstetter, E. 1999. *Topics in Microeconomics: Industrial Organizations, Auctions, and Incentives*. Cambridge University Press.

Wright, T.P. 1936. Factors affecting the cost of airplanes.*Journal of Aeronautical Sciences* **3**(4) 122-128.

WSJ. 2010. China Considers Further Rare-Earth Quotas. *Wall Street Journal* (December 29).

Yano, C. A., H. L. Lee. 1995. Lot Sizing with Random Yields: A Review. *Operations Research*, **43**(2), 311–334.

Yelle, L. E. 1979. The learning curve: Historical review and comprehensive survey. *Decision Science* **10** (2) 302-328.

Zhang, J. 2005. Transshipment and Its Impact on Supply Chain Members' Performance. *Management Science*, **51**(10), 1534–1539.

VITA

Tao Li was born in Yangquan, P. R. China, to Mr. Jintang Li and Ms. Aihua Feng. He received his Bachelor of Engineering degrees in Polymer Materials and Engineering and Financial Management, Master of Science degree in Management Science and Engineering from Tianjin University, Tianjin, P. R. China. In Fall 2007, he entered the Ph.D. program in Operations Management at the University of Texas at Dallas (UTD). While he was pursuing his Ph.D. degree, he received his Master of Business Administration degree in 2011 and Master of Science degree in Supply Chain Management in 2012 from UTD. At UTD, he taught the undergraduate core course in Operations Management in Spring 2011, Summer 2011, Spring 2012, and Summer 2012.